THE MIXING OF SCALAR MESONS AND THE POSSIBLE NONSTRANGE DIBARYONS

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The mixing of scalar mesons is an open problem. In this work, the idea of the mixing of scalar mesons will be applied to dibaryon system on quark level. By introducing the mixing of scalar mesons, we dynamically investigate the structure of possible nonstrange dibaryons in the chiral $SU(3)$ quark model by solving the resonating group method equation. The results show that no matter what kind of mixing is considered, the binding energies of nonstrange dibaryons would become stable if reasonable parameters are used.

Keywords: Quark model; chiral symmetry; dibaryon, the mixing of scalar mesons.

1. Introduction

The mixing of scalar mesons is an open problem, since the structure of scalar meson is unclear and controversial.1 Now we will apply the idea of the mixing of scalar mesons to dibaryon system.

There are many works to study the dibaryon system.2–11 In this work, we will focus the nonstrange dibaryon system on quark level. In 1987, Yazaki analyzed systems with two nonstrange baryons in the framework of the cluster model. The author considered the one-gluon exchange (OGE) interaction and the confining potential between two quarks and showed that $\Delta\Delta$ system could be bound state because the color magnetic interaction between two clusters is attractive.2 In the quark delocalization model, the authors studied the structure of the $\Delta\Delta$ system and found that this system is a deeply bound state.3–5 In the chiral $SU(3)$ quark model,6 the authors also studied this system and found this system is also a bound state,8 but the binding energy is not as large as those in the quark delocalization model. However, the mixing of scalar mesons is never considered for dibaryon systems in above works. Therefore, in the present work, we would like to further study these nonstrange dibaryon systems.

Among the quark models, one of the most successful models is the chiral $SU(3)$ quark model.6 In this model, the source of the constituent quark mass can
be logically explained from the underlying chromodynamics (QCD) theory of the strong interaction. Since spontaneous vacuum breaking has to be considered, and as a consequence the coupling between the quark field and the Goldstone boson must be introduced to restore the chiral symmetry. In this sense, the chiral quark model can be regarded as a quite reasonable and useful model to describe the medium-range nonperturbative QCD effect. This model has been quite successful in reproducing the energies of the baryon ground states, the binding energy of deuteron, the nucleon-nucleon (NN) scattering phase shifts of different partial waves, and the hyperon-nucleon (YN) cross sections by solving the resonating group method (RGM) equation.

Recently, using this chiral SU(3) quark model, the author firstly introduced the mixing of scalar mesons into NN system\textsuperscript{12} and YN system.\textsuperscript{13} In Ref.\textsuperscript{12} showed that no matter what kind of mixing is taken, the scattering phase shifts of nucleon-nucleon system can be reasonably reproduced in the chiral SU(3) quark model. Inspired by this, by introducing the mixing of scalar mesons and using the same set of parameters used in NN scattering processes,\textsuperscript{12} we would like to further investigate the structure of possible nonstrange dibaryons.

2. Formulation

The chiral SU(3) quark model has been described in the literature\textsuperscript{6} and we refer the reader to the work for details. Here we give the salient feature of this model. In the chiral SU(3) quark model, the coupling between chiral field and quark is introduced to describe nonperturbative QCD effect. The interacting Lagrangian can be written as:

\[ L_I = -g_{ch} \overline{\psi} \left( \sum_{a=0}^{8} \sigma^a \lambda_a + i \sum_{a=0}^{8} \pi^a \lambda_a \gamma_5 \right) \psi, \]  

where \( \lambda_0 \) is a unitary matrix, \( \sigma_0, \ldots, \sigma_8 \) are the scalar nonet field, and \( \pi_0, \ldots, \pi_8 \) the pseudoscalar nonet fields. The \( L_I \) is invariant under the infinitesimal chiral SU(3)\(_L \times SU(3)\(_R \) transformation, and only one coupling constant \( g_{ch} \) is needed by chiral symmetry requirement.

The total hamiltonian of baryon-baryon systems can be written as:

\[ H = \sum_{i=1}^{6} T_i - T_G + \sum_{i<j=1}^{6} V_{ij}, \]  

\[ V_{ij} = V_{ij}^{\text{conf}} + V_{ij}^{\text{GGE}} + V_{ij}^{\text{ch}}, \]

where \( \sum_i T_i - T_G \) is the kinetic energy of the system, and \( V_{ij} \) includes all interactions between two quarks. \( V_{ij}^{\text{conf}} \) is the confinement potential taken as quadratic
form, $V_{ij}^{OGE}$ is the OGE interaction, and $V_{ij}^{ch}$ represents the chiral fields induced effective quark-quark potential, which includes the scalar boson exchanges and the pseudoscalar boson exchange,

$$V_{ij}^{ch} = \sum_{a=0}^{8} V_{ij}^{\sigma_a} + \sum_{a=0}^{8} V_{ij}^{\pi_a}. \quad (4)$$

The detailed formula expressions can be found in Ref.

The definition of mixing of scalar mesons is:

$$\sigma = \sigma_8 \sin \theta_s + \sigma_0 \cos \theta_s,$$
$$\epsilon = \sigma_8 \cos \theta_s - \sigma_0 \sin \theta_s. \quad (5)$$

here the mixing angle of scalar singlet and octet mesons is $\theta_s$. Three cases will be discussed: the first is for no mixing, where the mixing angle is zero; the second is general mixing where the mixing angle is $\theta_s = -18^\circ$ introduced in Ref., based on investigation of a dynamically spontaneous symmetry breaking mechanism; and the third is ideal mixing where the mixing angle is $35.3^\circ$, which means that $\sigma$ only acts on the $u(d)$ quark, and $\epsilon$ meson on the $s$ quark.

Now we briefly give the procedure for the parameters determination. The initial input parameters are taken to be the usual values: i.e, the harmonic oscillator width parameter $b_u = 0.5$ fm, the up (down) quark mass $m_u (d) = 313$ MeV, the coupling constant for scalar and pseudoscalar chiral field coupling, $g_{ch}$, is fixed by the relation

$$\frac{g_{ch}^2}{4\pi} = \frac{9 m_u^2 g_{NN\pi}^2}{25 M_N^2 4\pi}, \quad (6)$$

with the experimental value $g_{NN\pi}^2/4\pi = 13.67$. The mass of the mesons are taken to be experimental values, except for the $\sigma$ meson. The cutoff mass is taken to be the value close to the chiral symmetry breaking scale. The OGE coupling constants $g_{ch}$ and the strengths of the confinement potential $a_{uu}$ and $a_{00}$ are determined by baryon masses and their stability conditions. The model parameters are listed in Table 1.

3. Resonating group method (RGM)

Resonating group method is a well established method for studying the interaction between two clusters.

The total, antisymmetrized six-quark wave function in orbital, spin, flavor and color space is of the following form:

$$\Psi = \mathcal{A}[\phi_A(\xi_1, \xi_2)\phi_B(\xi_3, \xi_4)\psi(R_{AB})Z(R_{CM})]_{STC}. \quad (7)$$
Table 1. Model parameters. Meson masses and cutoff masses: \( m_{\pi} = 138 \text{ MeV}, m_K = 495 \text{ MeV}, m_{\eta} = 549 \text{ MeV}, m_{\eta'} = 957 \text{ MeV}, m_{\sigma'} = m_{\epsilon} = m_{\kappa} = 980 \text{ MeV}, \Lambda = 1100 \text{ MeV} \) for all mesons.

<table>
<thead>
<tr>
<th>Set</th>
<th>( \theta_s )</th>
<th>( b_s (\text{fm}) )</th>
<th>( m_u (\text{MeV}) )</th>
<th>( g_{ch} )</th>
<th>( m_{\sigma} (\text{MeV}) )</th>
<th>( g_u^2 )</th>
<th>( a_{uu} (\text{MeV}/\text{fm}^2) )</th>
<th>( a_{0 uu} (\text{MeV}/\text{fm}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0°</td>
<td>0.5</td>
<td>313</td>
<td>2.621</td>
<td>594</td>
<td>0.785</td>
<td>48.1</td>
<td>-43.6</td>
</tr>
<tr>
<td>II</td>
<td>-18°</td>
<td>0.5</td>
<td>313</td>
<td>2.621</td>
<td>491</td>
<td>0.785</td>
<td>49.0</td>
<td>-44.9</td>
</tr>
<tr>
<td>III</td>
<td>35°</td>
<td>0.5</td>
<td>313</td>
<td>2.621</td>
<td>559</td>
<td>0.785</td>
<td>50.0</td>
<td>-46.0</td>
</tr>
</tbody>
</table>

where \( \xi_1, \xi_2 \) are the internal coordinates for the cluster A, and \( \xi_3, \xi_4 \) are the internal coordinates for the cluster B. \( R_{AB} \) is the relative coordinate between \( A \) and \( B \), and \( R_{CM} \) is the center of mass coordinate of the total system. \( S \) and \( T \) denote the total spin and isospin of the cluster A and cluster B, and the \( \phi_A \) and \( \phi_B \) are the antisymmetrized wave functions of cluster A and B, respectively. The \( \chi(R_{AB}) \) is the relative wave function of the two clusters, and \( Z(R_{CM}) \) is the total center of mass wave function of the six-quark state which can be chosen freely due to the Galilei invariance of the system. The symbol \( \mathcal{A} \) is the antisymmetrizing operator defined as

\[
\mathcal{A} \equiv 1 - \sum_{i \in A, j \in B} P_{ij},
\]

where \( P_{ij} \) is the permutation operator of the \( i \)th and \( j \)th quarks.

Substituting \( \Psi \) into the projection equation

\[
\langle \delta \Psi | (H - E) | \Psi \rangle = 0,
\]

we obtain the coupled integro-differential equation for the relative function \( \chi \) as

\[
\int \left[ \mathcal{H}(R, R') - EN(R, R') \right] \chi(R') dR' = 0,
\]

where the Hamiltonian kernel \( \mathcal{H} \) and normalization kernel \( N \) can, respectively, be calculated by

\[
\begin{align*}
\left\{ \mathcal{H}(R, R') \right\} &= \left\{ [\hat{\phi}_A(\xi_1, \xi_2) \hat{\phi}_B(\xi_3, \xi_4)] \delta(R - R_{AB}) \right. \\
N(R, R') &= \left\{ \left[ \mathcal{A} \left[ \hat{\phi}_A(\xi_1, \xi_2) \hat{\phi}_B(\xi_3, \xi_4) \right] \delta(R' - R_{AB}) \right] \right. \\
&= \left. \left( \mathcal{H}(R, R') \right) \mathcal{A} \left[ \hat{\phi}_A(\xi_1, \xi_2) \hat{\phi}_B(\xi_3, \xi_4) \right] \delta(R' - R_{AB}) \right)
\end{align*}
\]

(11)
Eq. (11) is the so-called coupled-channel RGM equation. Expanding unknown $\chi(R_{AB})$ by employing well-defined basis wave functions, such as Gaussian functions, one can solve the coupled-channel RGM equation for a bound-state problem to obtain the binding energy for the two-cluster systems. The details of solving the RGM equation can be found in Refs. 19–23.

4. Result

Now three different states will be selected. One state is $NN$ system with $L = 0, S = 1, T = 0$, this is the only dibaryon state confirmed by experiment, we call it deuteron. Other two states are from $\Delta\Delta$ system: one is the state with $L = 0, S = 0, T = 3$, in which only the central force is needed; and another state is called deltaron, $L = 0, S = 3, T = 0$, in which the hidden color channel ($CC$) is included.

$$|CC\rangle = -\frac{1}{2}|\Delta\Delta\rangle + \frac{\sqrt{5}}{2}A_{STC}|\Delta\Delta\rangle$$

where $A_{STC}$ stands for the antisymmetrizer in the spin-isospin-color space.

Using the model parameters listed in Table 1, we dynamically investigate the structure of possible nonstrange dibaryons in the chiral $SU(3)$ quark model by solving the RGM equation. These model parameters can give a good description of the energies of the baryon ground states, the binding energy of deuteron, and the experimental data of the nucleon-nucleon scattering processes. The calculated binding energies are listed in the Table 2 for three different cases. From Table 1 We can see that the mass of $\sigma$ is somewhat different for these three cases, because it is decided by fitting the deuteron experimental data of 2.22 MeV, the calculated values are shown in Table 2. For no mixing ($\theta^* = 0^\circ$), when $m_\sigma$ is taken to be 594 MeV, the binding energy of the deuteron is 2.21 MeV. For $\theta^* = -18^\circ$ mixing, when $m_\sigma$ is taken to be 491 MeV, the binding energy of the deuteron is 2.18 MeV. For the ideal mixing ($\theta^* = 35.3^\circ$), when $m_\sigma$ is taken to be 559 MeV, the binding energy of the deuteron is 2.19 MeV. From Table 2, we see that the binding energy for $\Delta\Delta$ ($ST = 03$) is about 22 MeV, and not changed much with the different
mixing. Similarly, for $\Delta\Delta (S T = 30)$ state, the binding energy is very stable deeply bound state.

In summary, by introducing the mixing of scalar mesons, we dynamically investigate the structure of possible nonstrange dibaryons in the chiral $SU(3)$ quark model by solving the resonating group method equation. The results show that no matter what kind of mixing is considered, the binding energies of nonstrange dibaryons would become stable if reasonable parameters are used.

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