WOBBLING ROTATION IN ATOMIC NUCLEI *

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The wobbling states are described microscopically with the Triaxial Projected Shell Model (TPSM), and the experimental wobbling bands firmly established in \( ^{163}\text{Lu} \) have been well reproduced by the present calculation. By using calculated wave functions of wobbling states, the projections of the total angular momentum along the three body-fixed axes are calculated as functions of spin. This calculation results in the dynamic geometry of the angular momentum in the intrinsic frame, and thus promises to provide insight into the wobbling motion in nuclei. The similar TPSM calculations have been performed to study the possible wobbling excitation in the neighboring nucleus \( ^{164}\text{Lu} \). It has been found that the wobbling structure in odd-odd Lu may be modified by the extra neutron, but not completely destroyed, indicating the possibility for observing the wobbling bands in the odd-odd Lu isotopes.

1. Introduction

More general schemes of rotational motion suggested by classical mechanics are precession and wobbling,\(^1\) henceforth both are called wobbling. The wobbling motion in nuclei is the excitation mode associated with the rotational asymmetry of a triaxial quantum system, which was proposed first by Bohr and Mottelson in the early 1970s.\(^2,3\) It is expected that in a rotating stable triaxial nucleus the angular momentum is not aligned with any of the body-fixed axes and rather processes and wobbles around one of these axes in a manner just like that of an asymmetric top. Although the wobbling rotation is expected to be a general excitation mode that would occur in triaxially deformed nuclei, it had never been realized in experimental nuclear spectra until the clear evidence was found for \( ^{163}\text{Lu} \) in 2001. The nuclear

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wobbling mode has been firmly established by the crucial experiments that discovered the first- and second-phonon wobbling bands in \(^{163}\)Lu.\(^{4-6}\) The subsequent experiments for searching other wobblers have been performed quickly over the past decade and the wobbling bands have been seen in only a few Lu isotopes, namely, in \(^{161}\)Lu,\(^{7}\) \(^{165}\)Lu\(^{8,9}\) and \(^{167}\)Lu.\(^{10}\) In spite of many experiments, e.g., in Refs.\(^{11-13}\) searching for wobbling modes in nearby nuclei there are no more further examples of wobbling bands. Only very recently, the wobbling bands have been observed for the first time in a nucleus other than Lu, namely, in \(^{167}\)Ta, making this collective triaxial rotational mode a more general phenomenon.\(^{14}\)

It is an essential point in the study of nuclear rotations to establish whether and how the collective wobbling motion is generated microscopically by the nucleons. One of best approaches to provide insight into this quantum many-body problem is the shell model. It is likely that the wobbler \(^{163}\)Lu may serve as the best target nucleus for such a theoretical study since its rotational spectra have been measured up to the second phonon band and the configurations of the wobbling bands well assigned, which provide a complete information about the phonon energies and the intrinsic structure. In the present paper, the wobbling states are described microscopically by the Triaxial Projected Shell Model (TPSM). We will see that the experimental wobbling bands observed in \(^{163}\)Lu can be well reproduced by the present calculation. We also show that a deep understanding of the wobbling motion in nuclei can be achieved by analyzing the dynamic geometry of the angular momentum in the intrinsic frame, which can be calculated by using the TPSM wave functions. To investigate the possible wobbling excitation in odd-odd Lu nuclei the similar TPSM calculation has been performed for the neighboring nucleus \(^{164}\)Lu, in which an extra neutron is added to a wobbler \(^{163}\)Lu. It has been found that the wobbling structure may be modified by the addition of the extra neutron in some degrees, but not completely destroyed, indicating the possibility to observe the wobbling bands in the odd-odd Lu isotopes.

The model used is briefly described in Section 2. The TPSM calculations for the wobbling bands and discussions are given in Section 3. General conclusions are summarized in Section 4.

2. Brief description of the model

The theory incorporates the many-body correlations into the mean-field approximations to make shell-model calculations possible for heavy nuclei. The present model employs a triaxially deformed basis and constructs the
model space by including multi-quasi-particle (qp) states and performs exact three-dimensional angular momentum projection. A realistic two-body Hamiltonian is then diagonalized in this space. The theory is briefly described below and the more details can be found in Ref. 16. In the TPSM, the trial wave function may be written as
\[ |Ψ_{IM}⟩ = \sum_{Kκ} f^K_{IK} \hat{P}^I_{MI} |Φ_κ⟩, \]  
where \( \hat{P}^I_{MI} \) is the three-dimensional angular-momentum-projection operator,
\[ \hat{P}^I_{MI} = \frac{2I + 1}{8\pi^2} ∫ dΩ D^I_{MK}(Ω) \hat{R}(Ω), \]
where \( R(Ω) \) is the rotation operator which has the explicit form, \( e^{−iα J_z} e^{−iβ J_z} e^{−iγ J_z} \). The \( σ \) in Eq. (1) specifies the states with the same angular momentum \( I \). The dimension of the summation in Eq. (1) is \( K × κ \), where \( |K| ≤ I \) and \( κ \) is usually in the order of \( 10^2 \). The \(|Φ_κ⟩\) represents a set of multi-qp states associated with the triaxially deformed qp vacuum \(|0⟩\). For odd proton nuclei included is the set of 1-, 3-, and 5-qp states,
\[ \{ α^+_1 |0⟩, α^+_2 α^+_1 |0⟩, α^+_2 α^+_1 α^+_2 α^+_1 |0⟩ \}. \]
By carrying out the variational procedure with respect to the wave function, precisely the coefficients \( f^K_{IK} \), we obtain the eigenvalue equation,
\[ ∑_{Kκ} f^K_{IK} \left( ⟨Φ_κ|HP^I_{K′K}|Φ_κ⟩ - E⟨Φ_κ|P^I_{K′K}|Φ_κ⟩ \right) = 0. \]

The shell model Hamiltonian considered involves a large number of nucleons moving in a spherical Nilsson potential and an interaction of separable multipole \( Q \cdot Q \) plus monopole pairing plus quadrupole pairing,
\[ H = H_0 - \frac{1}{2} ∑_{λ=2}^{4} ∑_{μ=−λ}^{λ} Q^λ_{μ} P^λ_{μ} - G_0 P^0_0 P^0_0 - G_2 ∑_{μ=−2}^{2} P^μ_2 P^μ_2. \]  
In Eq. (5), \( \hat{H}_0 \) is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force. The second term is quadrupole-quadrupole (\( QQ \)) interaction that includes the nn, pp, and np components. The \( QQ \) interaction strength \( χ \) is determined in a self-consistent way to match the
quadrupole deformation used in construction of the qp basis. The third term in Eq. (5) is monopole pairing, whose strength $G_M$ is of the standard form $G/A$, with $G = 17.93$ MeV for both protons and neutrons, which approximately reproduces the observed odd-even mass differences in this mass region. The last term is quadrupole pairing, with the strength $G_Q=0.12 G_M$ as usual.

The triaxially deformed single particle states are generated by the Nilsson Hamiltonian

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3} \hbar \omega \epsilon_2 \left( \cos \gamma \hat{Q}_0 - \sin \gamma \frac{\hat{Q}_{+2} + \hat{Q}_{-2}}{\sqrt{2}} \right), \tag{6}$$

where the parameters $\epsilon_2$ and $\gamma$ describe quadrupole deformation and triaxial deformation, respectively.

3. Results of calculations and discussion

The three major shells of $N = 4, 5, 6$ for both neutrons and protons are included to calculate the Nilsson single particle states. The deformation parameters used to generate the deformed quasiparticle basis states are $\epsilon_2 = 0.38$, $\gamma = 20^\circ$ and $\epsilon_4 = 0.0$ for $^{163}$Lu, which are in consistent with the TRS calculation. The shell model basis contains the multi-quasiparticle configurations up to the 5-qp states of $2n3p$, which has a seniority quantum number high enough to describe the rotational bands up to very high spin $I \sim 101/2$, the highest spin observed in $^{163}$Lu. The calculated wobbling bands of $^{163}$Lu up to second phonon states are shown in Figure 1, and compared with the experimental data. In Figure 1, the excitation energy minus a rigid-rotor reference is plotted as a function of spin for the triaxial strongly formed bands, TSD1, TSD2 and TSD3, which are identified as zero phonon, one phonon and two phonon wobbling bands, respectively. The collective wobbling behavior can be described within a phonon model, where the energy of each band is expressed as $E = \frac{\hbar^2}{2} I(I+1) + \hbar \omega_{\text{wob}} (n_\omega + 1/2)$, where the $n_\omega$ is phonon quantum number and $\omega_{\text{wob}}$ is the wobbling frequency that measures the wobbling phonon excitation energy. It is seen from figure 1 that the experimental energies of the all three bands of the wobbling family are well reproduced by the present calculations. The experimental excitation energies of the one phonon band and the two phonon band relative to the zero phonon band, and thus the wobbling frequencies $\omega_{\text{wob}}$, have been also well reproduced by these calculations. It is shown that the observed excitation energy of the TSD3 band is not twice as large as the excitation energy of the TSD2 band in the range of observed angular
momentum, indicating the anharmonicity of the wobbling excitation. The presence of the anharmonicity implies the existence of the complex coupling between the collective wobbling motion and the single quasiparticle excitations in a rapidly rotating triaxial nucleus. Nevertheless, the effect of anharmonicity on the wobbling excitation energy has been well described by the TPSM theory. The anharmonicity of the wobbling motion in the $E2$ transitions was studied in detail by using the particle-rotor model of one $i_{13/2}$ quasiparticle coupled to the core of triaxial shape.

It is an advantage of the TPSM theory that the wave function in the form of angular momentum projection can be used conveniently to calculate the dynamic geometry of the total angular momentum in the intrinsic frame. Such a calculation is particularly useful in understanding the wobbling motion in nuclei because the family of wobbling bands is characterized with the common similar intrinsic structure but the different tilting of the angular momentum vector with respect to one of body-fixed axes. In Figure 2, presented are the expectation values of $I^x$, $I^y$ and $I^z$, divided by $I(I+1)$, as functions of spin for the lowest two TSD bands in $^{163}$Lu, calculated by using the TPSM wave functions which reproduce the experimental band energies as shown in Figure 1. The TSD1 and TSD2 are the signature partner bands and their spin sequences can be expressed by means of the signature $\alpha$ as $I = 2n + \alpha$ with the favored signature $\alpha=1/2$ and the unfavored signature $\alpha=-1/2$, respectively. It is seen from Figure 2 that the favored

Fig. 1. Excitation energy minus a rigid-rotor reference for denoted TSD bands in $^{163}$Lu. The inertia parameter was set to 7.5 keV/$h^2$ and the TSD1 band head at $I = 13/2$ was set to $E_{exc}=1.723$ MeV. The calculated results (open symbols) are compared with the experimental data (solid symbols).
Fig. 2. Expectation values of $I_x^2$, $I_y^2$ and $I_z^2$, divided by $I(I+1)$, as functions of spin for the lowest two TSD bands in $^{163}$Lu, calculated by using the TPSM wave functions that reproduce the experimental band energies as shown in Fig. 1.

signature band TSD1, $I=13/2$, 17/2, 21/2, ..., has the angular momentum vector most closing to the x-axis, the shortest body-fixed axis, and thus the rotational band is energetically lowest and assigned as the $n_w=0$ phonon state. While in the unfavored signature band TSD2, $I=15/2$, 19/2, 23/2, ..., the angular momentum vector lies farther from the x-axis and thus the band is relatively higher in energy and assigned as the $n_w=1$ phonon state. The calculated wave functions show a very similar structure for the wobbling bands, namely, the $i_{13/2}$ proton qp-configuration dominates in a large range of spin. The very similar intrinsic structure together with the tilting motion of the angular momentum vector, as being demonstrated in Figure 1, characterize the wobbling mode in nuclei. The stable triaxial deformation and the high-$j$ and low-$K$ orbit, like $\pi[660]l/2$ in $^{163}$Lu, play an essential role in the formation of the wobbling bands. For the wobbling excitation the crucial contribution from the aligned particle was also born out from the calculation of the wobbling mode in $^{163}$Lu by means of the cranking shell model plus random phase approximation.$^{19}$

To date no any experimental evidence for the wobbling mode has been found in odd-odd nuclei. To investigate this possibility the similar TPSM calculation has been performed for nucleus $^{164}$Lu, in which an extra neutron is added to the wobbler $^{163}$Lu. The deformation parameters used to generate the deformed quasi-particle basis states are $\varepsilon_2=0.41$, $\gamma=21^\circ$ and $\varepsilon_4=0.0$, which are in consistent with the TRS calculation. With these deformations the calculated yrast TSD band is in agreement with the experimental TSD1
band in $^{164}$Lu. In order to make the result more instructive the calculation of the geometry of the angular momentum has been performed for the single 2qp configuration of $\pi[660]1/2\nu[523]5/2$, a proton in the $i_{13/2}$ and a neutron in the $f_{7/2}$ shells, which dominates the TPSM wave functions in a large range of spin. Calculated expectation values of $I_x^2$, $I_y^2$ and $I_z^2$, divided by $I(I+1)$, as functions of spin for the lowest two TSD bands in $^{164}$Lu are shown in Figure 3. The characteristic wobbling motion remains in $^{164}$Lu as it is shown in Figure 3 that in going from the $n_{\omega}=0$ band, $I=10, 12, 14, ...$, to the $n_{\omega}=1$ band, $I=11, 13, 15, ...$, the angular momentum vector lies progressively farther from the $x$-axis, just as do in the case of $^{163}$Lu. It seems to be reasonable to expect that the wobbling mode presented in odd-$A$ Lu nuclei may be modified by the addition of the extra neutron but not completely destroyed, indicating the possibility for the observation of the wobbling bands in the odd-odd Lu isotopes.

4. Summary

The wobbling rotation in atomic nuclei can be described full quantum-mechanically by mean of the TPSM theory. The wobbling bands observed in $^{163}$Lu have been well reproduced by the present calculation up to very high spins and the second wobbling phonon excitations within the shell model.
space that spans the set of 1qp, 3qp and 5qp states. The wobbling motion has been vividly demonstrated with the dynamic geometry of the angular momentum in the body-fixed frame, which may be calculated by using the TPSM wave functions. The wobbling excitation is characterized with the common similar intrinsic structure, particularly the stable triaxial shape, and the tilting motion of the angular momentum vector in the intrinsic frame where the vector lies progressively farther from the shortest $x$-axis when going from the zero phonon to the higher phonon numbers. Once these conditions are fulfilled there exist a possibility for the observation of new wobbling nuclei, for example, $^{164}$Lu is a candidate.

References