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IBM-1模型中的随机相互作用

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Outline

- 1.Introduction of random interactions
- 2.IBM model with random interactions
- 3.Correlations of low-lying states
- 4. Arguments in terms of the U(5) limit
- 5. Summary and conclusions

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Random Interactions

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Orderly Spectra from Random Interactions

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We investigate the low-lying spectra of many-body systems with random two-body interactions, specifying that the ensemble be invariant under particle-hole conjugation. Surprisingly we find patterns reminiscent of more orderly interactions, such as a predominance of $J = 0$ ground states separated by a gap from the excited states, and evidence of phonon vibrations in the low-lying spectra. [S0031-9007(98)05710-X]

PACS numbers: 05.30.–d, 05.45.+b, 21.60.Cs, 24.60.Lz

In 1998, Johnson, Bertsch, and Dean discovered that spin zero ground state dominance can be obtained by using random two-body interactions (Phys. Rev. Lett. 80, 2749)

0 g.s. predominance

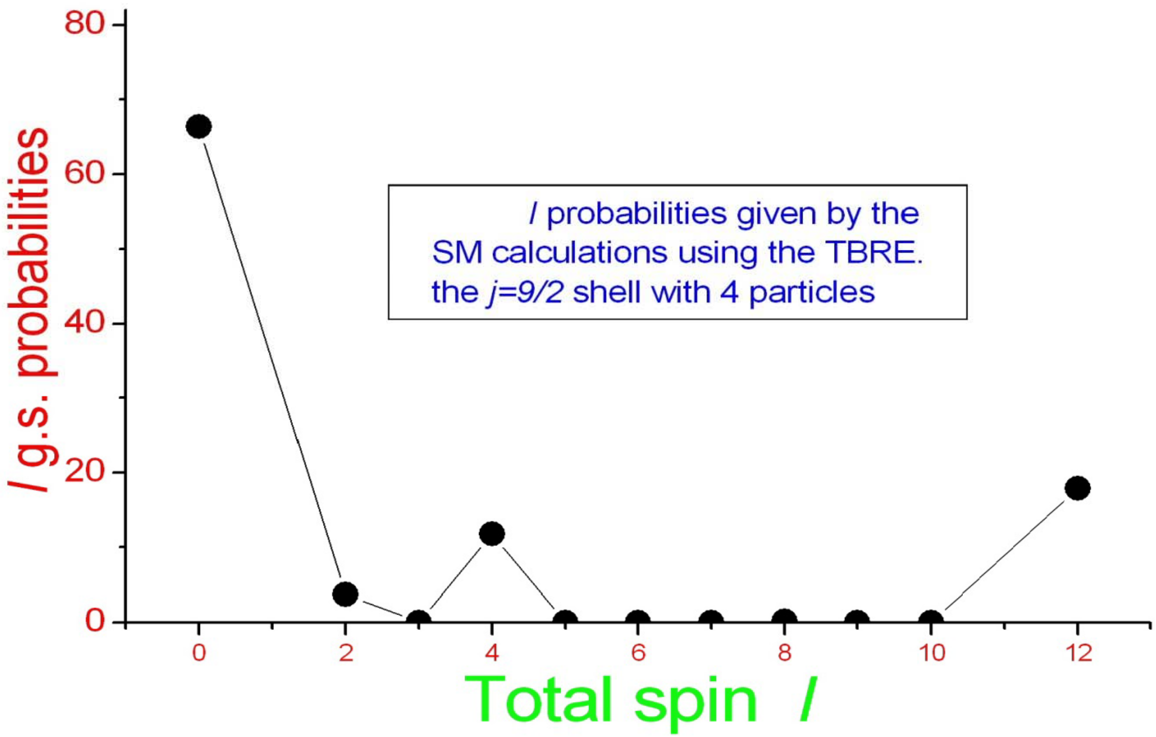
TABLE I. Percentage of ground states (g.s.) of the RQE that have $J = 0, T = T_z$ for our target nuclides, as compared to the percentage of all states in the model spaces that have these quantum numbers.

N	Ω	Nucleus	$J = 0, T = T_z$ g.s.	$J = 0, T = T_z$ Total space
6	12	^{22}O	76%	9.8%
6	20	^{46}Ca	75%	3.5%
$N = 4, Z = 4$	12	^{24}Mg	66%	1.1%

identical particles automatically have $T = T_z = 3$.) We see that between two-thirds and three-quarters of the spectra have the singlet state as the lowest. This is not a trivial consequence of the dimensionality of our model spaces,

0 g.s. predominance

$$j = \frac{9}{2}, \quad n = 4$$



Spin	Dimension
0	2
2	2
3	1
4	3
5	1
6	3
7	1
8	2
9	1
10	1
12	1

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Available Results

Empircal method **Zhao&Arima&Yoshinaga (2002)**

Mean-field method **Bijker-Frank (2003)**

Geometrid method **Chau et al. (2003)**

Spectral Radius **Papenbrock & Weidenmueller (2004)**

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Time reversal invariance (TRI) Zuker et al. (2002);

Time reversal invariance? Bijker&Frank&Pittel (1999);

Width ? Bijker&Frank (2000);

off-diagonal matrix elements for I=0 states Drozdz et al. (2001)

Highest symmetry &Time Reversal Otsuka&Shimizu(2004-2007)

Semi-empirical formula Shen, Zhao, Arima, and Yoshinaga(2006-2010)

The sd interacting boson model (IBM)

The sd interacting boson model (IBM) is a wonderful framework in nuclear structure theory (many thanks to its inventors, Prof. Arima and Prof. Iachello) with full of beauty of symmetry, simplicity, and rich structures exhibited in low-lying states of nuclei. That may be one of the key reasons for its popularity [experimentalists also use it]. It is (possibly) the simplest framework which can be designed in nuclear structure theory.

Because of its simplicity, it is an ideal tool to study the behavior of the random hamiltonian by using the IBM.

Collectivity in the IBM under random interactions

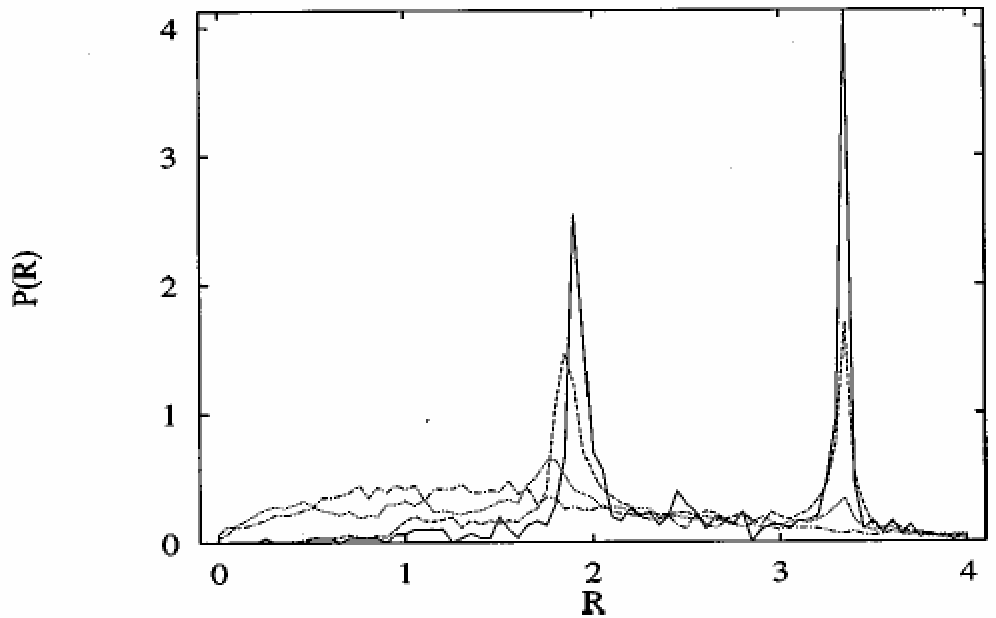

FIG. 2. Probability distributions $P(R)$ of the energy ratio R of Eq. (7) with $\int P(R)dR=1$ in the IBM with random two-body interactions for $N=3$ (dash-dotted), 6 (dotted), 10 (dashed), and 16 (solid).

Taken from PRC62, 014303(2000), by R. Bijker and A. Frank

$$R=\frac{E(4_1^+)-E(0_1^+)}{E(2_1^+)-E(0_1^+)},$$

R~2 for vibrational nuclei
R~3.3 for rotational nuclei

Tripartite classification

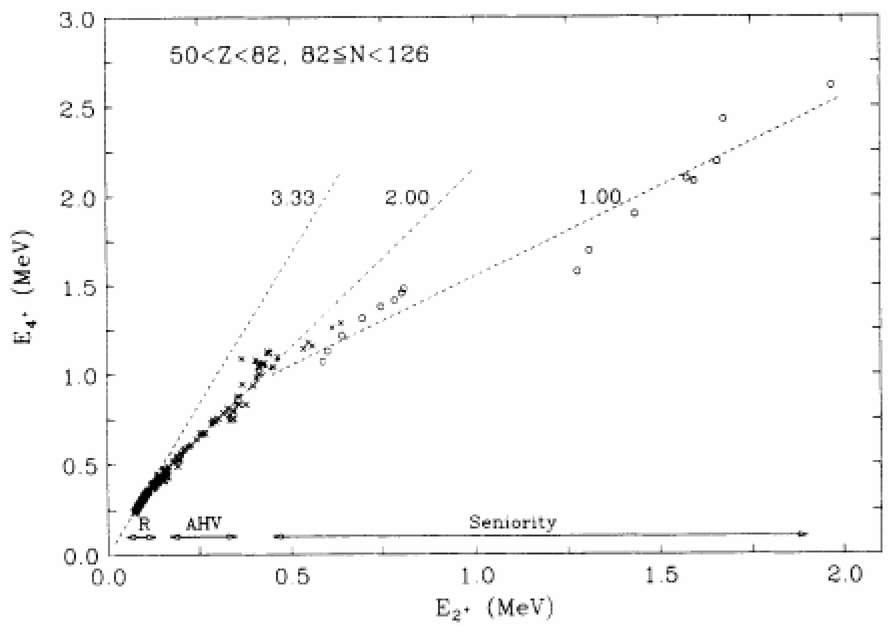

FIG. 3. A plot, for the $Z=50-82$, $N=82-126$ region, of $E(4_1^+)$ against $E(2_1^+)$ showing the division of the structural evolution into a tripartite classification of "seniority," anharmonic vibrator (AHV), and rotor (R) regions, corresponding to slopes of 1.0, 2.0, and 3.33. These regions are labeled at the bottom and demarcated by vertical dotted lines separated by narrow zones in which the structure varies rapidly in a way that resembles phase transitional behavior seen in other physical systems.

Taken from PRL72,3480(2000), by N. V. Zamfir and R. F. Casten

R~1 seniority
R~2 for vibrational nuclei
R~3.3 for rotational nuclei

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Correlation of low-lying states

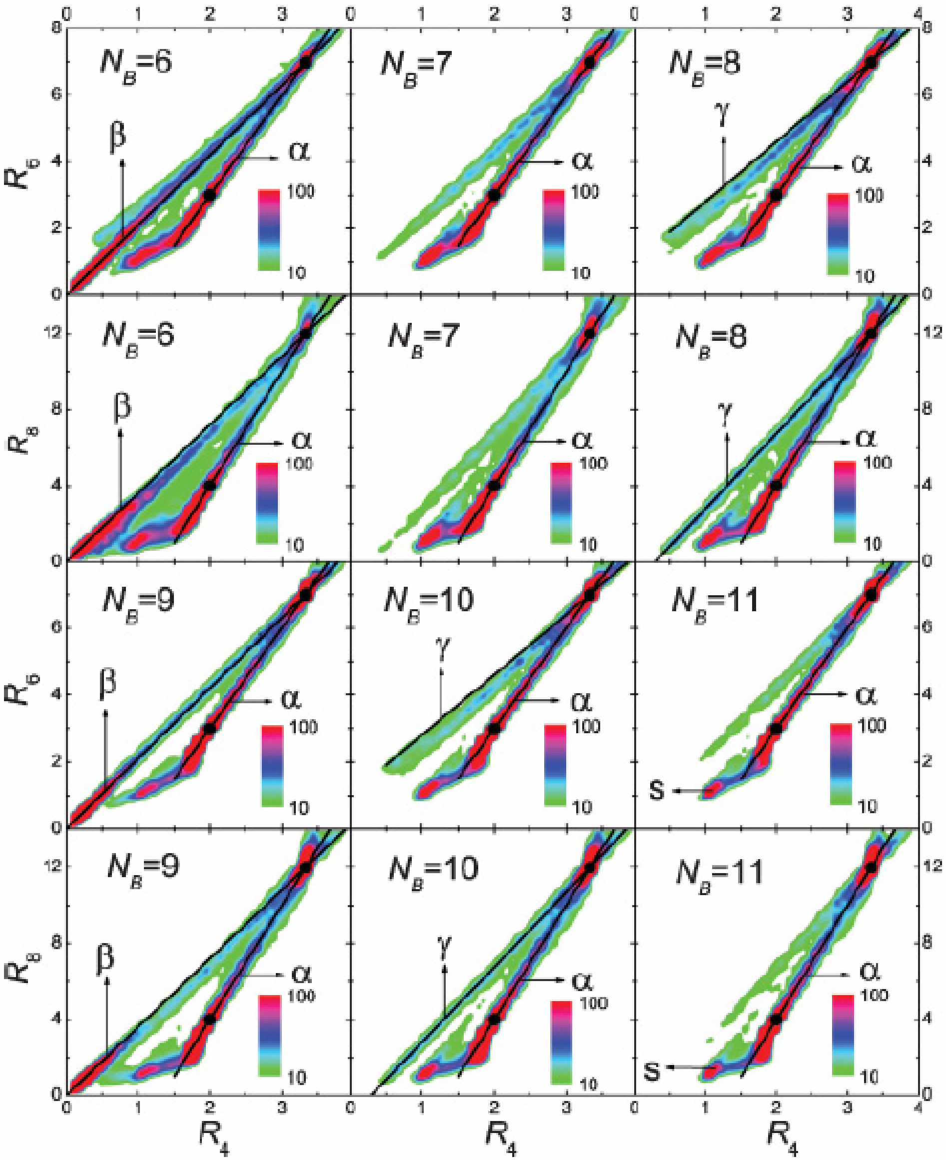


FIG. 1. (Color online) Distribution of R_I and R_4 for boson numbers $N_B = 6$–11. For each N_B, we perform the calculations by 60 000 sets of the TBRE. In addition to two very sharp vibrational and rotational peaks [denoted by solid black circles in each panel, i.e., $(R_4, R_6) = (2.0, 3.0)$ and $(3.3, 7.0)$, $(R_4, R_8) = (2.0, 4.0)$ and $(3.3, 12.0)$], three linear correlations between R_I $(I = 6, 8)$ and R_4 are easily noticed. We denote these new correlations by α, β, and γ. The slopes of solid lines corresponding to the α, β, and γ correlations are 3, $\frac{21}{10}$, and $\frac{9}{5}$ in the (R_4, R_6) plots; and they are 6, $\frac{18}{5}$ and $\frac{138}{35}$ in the (R_4, R_8) plots. See the text for details. Another relatively smaller "peak" at $R_I \simeq 1.0$ $(I = 4, 6, 8)$ is denoted by "S" (called the seniority-type correlation in Ref. [11]) for $N_B = 11$.

When g.s. spin=0, yrast states are highly correlated, even when interactions are random.

Taken from PRC. 83. 044302, by 雷杨 and 赵玉民

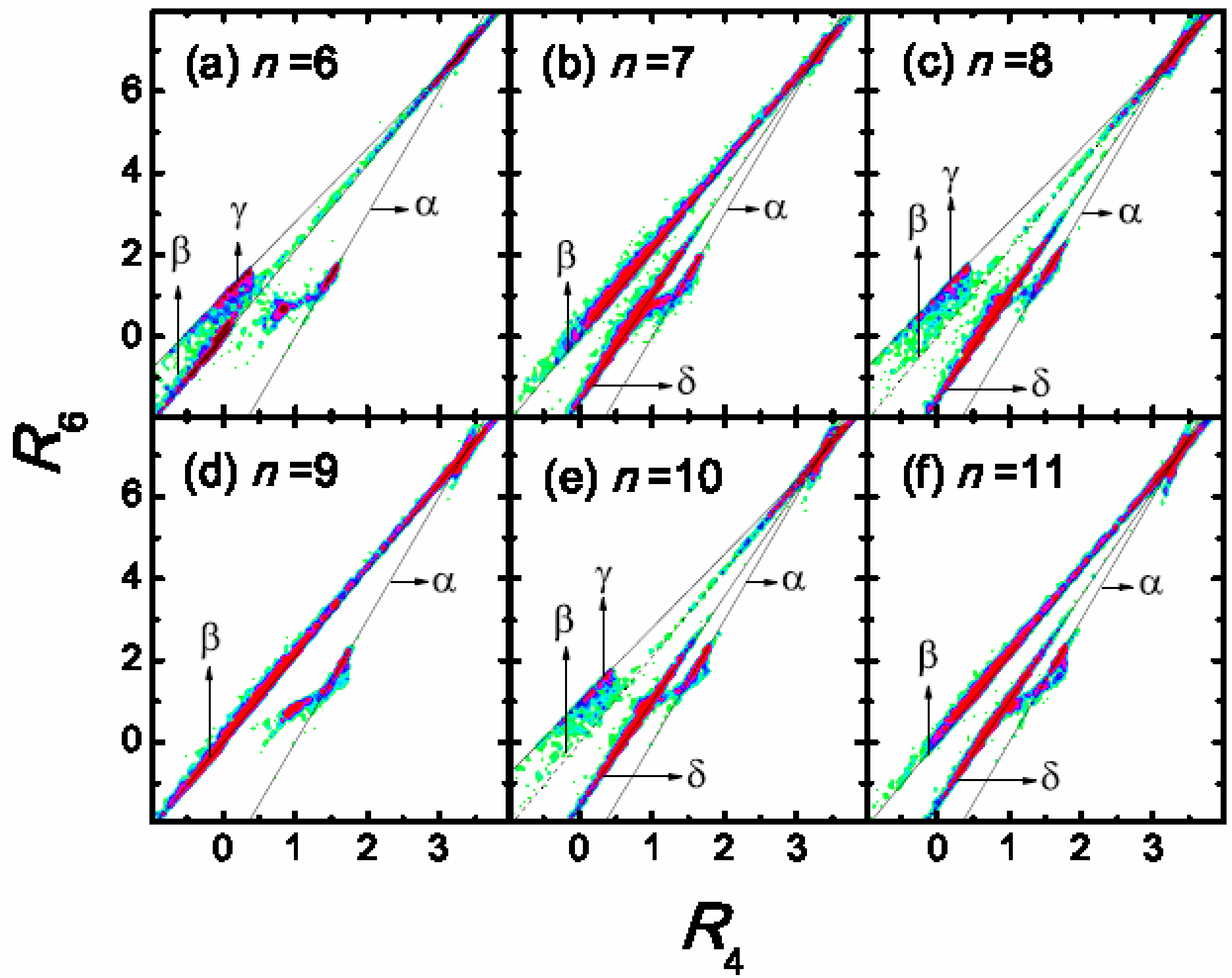
The IBM-1 model

Our *sd*-boson Hamiltonian is as follows:

$$H = \sum_{l_1 \le l_2; l_3 \le l_4; L} \frac{V^{(L)}_{l_1 l_2 l_3 l_4}}{\sqrt{(1+\delta_{l_1 l_2})(1+\delta_{l_3 l_4})}} [b^\dagger_{l_1} b^\dagger_{l_2}]^L [\tilde{b_{l_3}} \tilde{b_{l_4}}]^L, \tag{1}$$

where $l_i (i = 1, 2, 3, 4) = 0$ and 2, which represents the spin of s bosons and d bosons, respectively. $V^{(L)}_{l_1 l_2 l_3 l_4} = \langle l_1 l_2 : L | V | l_3 l_4 : L \rangle$ are two-body matrix elements. They follow the Gaussian distribution with average being zero and width being $\sqrt{(1+\delta_{l_1 l_2, l_3 l_4})}$. The ensemble of such defined coefficients are called the two-body random ensemble (TBRE).

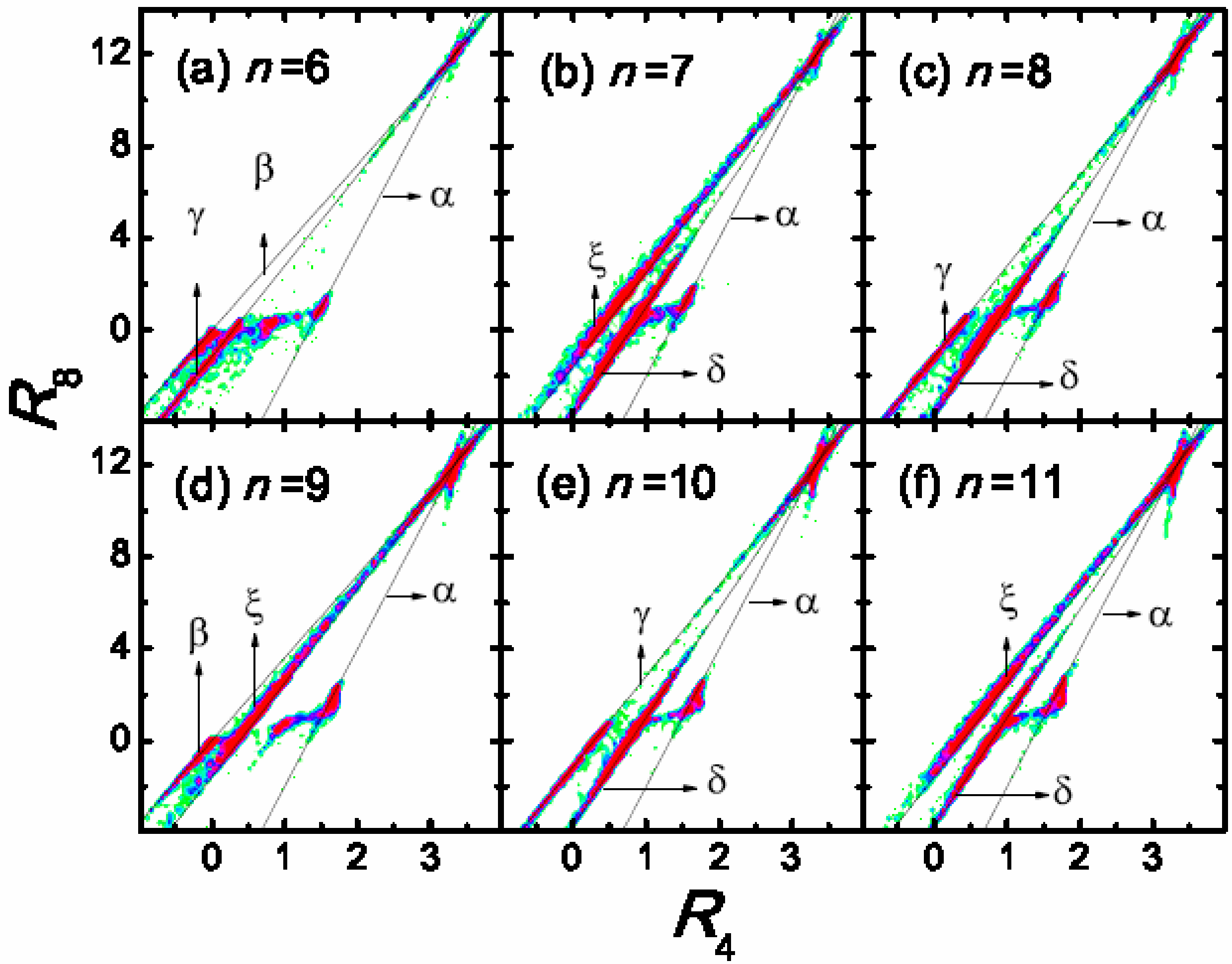
Correlation of low-lying states



g.s. not 0

Submitted to PRC, by 路毅 and 赵玉民

Correlation of low-lying states



g.s. not 0

U(5)极限

$$E_{\tau I} = C_1\tau(\tau+3) + C_2 I(I+1), \qquad (2)$$

where τ is the seniority of d bosons, I is the spin of the state, and

$$C_1 = \frac{-7V^{(0)}_{2222} + 10V^{(2)}_{2222} - 3V^{(4)}_{2222}}{70}, \quad C_2 = \frac{-V^{(2)}_{2222} + V^{(4)}_{2222}}{14}.$$

According to the reduction rule of the U(5) limit [17], for given boson number $n = 2v + \tau = 2v + 3n_\Delta + \lambda$. τ takes

$$\tau = n, \quad n-2, \quad \cdots, \quad 1 \text{ or } 0 \quad (n= \text{odd} \quad \text{or} \quad \text{even}).$$

The values of λ are

$$\lambda = \tau, \quad \tau - 3, \quad ..., \quad 0 \text{ or } 1, 2 \quad (\tau \quad \text{mod } 3).$$

Then total spin I of the system for each v and n_Δ is given by

$$I = \lambda, \quad \lambda + 1, \quad ..., \quad 2\lambda - 2, 2\lambda.$$

ber. Thus yrast spin-zero ground states of the U(5) limit with the requirement ($C_1 > 0, C_2 > 0$) correspond to $\tau = 3$ and $\lambda = 0$. In this case, the quantum numbers and eigen-energies of yrast spin I states are given by

$$\begin{aligned}
\tau &= 3, \quad \lambda = 0, \quad E_{0_1^+} = 18C_1, \\
\tau &= 1, \quad \lambda = 1, \quad E_{2_1^+} = 4C_1 + 6C_2, \\
\tau &= 3, \quad \lambda = 3, \quad E_{4_1^+} = 18C_1 + 20C_2, \\
\tau &= 3, \quad \lambda = 3, \quad E_{6_1^+} = 18C_1 + 42C_2, \\
\tau &= 5, \quad \lambda = 5, \quad E_{8_1^+} = 40C_1 + 72C_2.
\end{aligned}$$

$$R_4 = \frac{10C_2}{3C_2 - 7C_1}, \quad R_6 = \frac{21C_2}{3C_2 - 7C_1}, \quad R_8 = \frac{11C_1 + 36C_2}{3C_2 - 7C_1}.$$

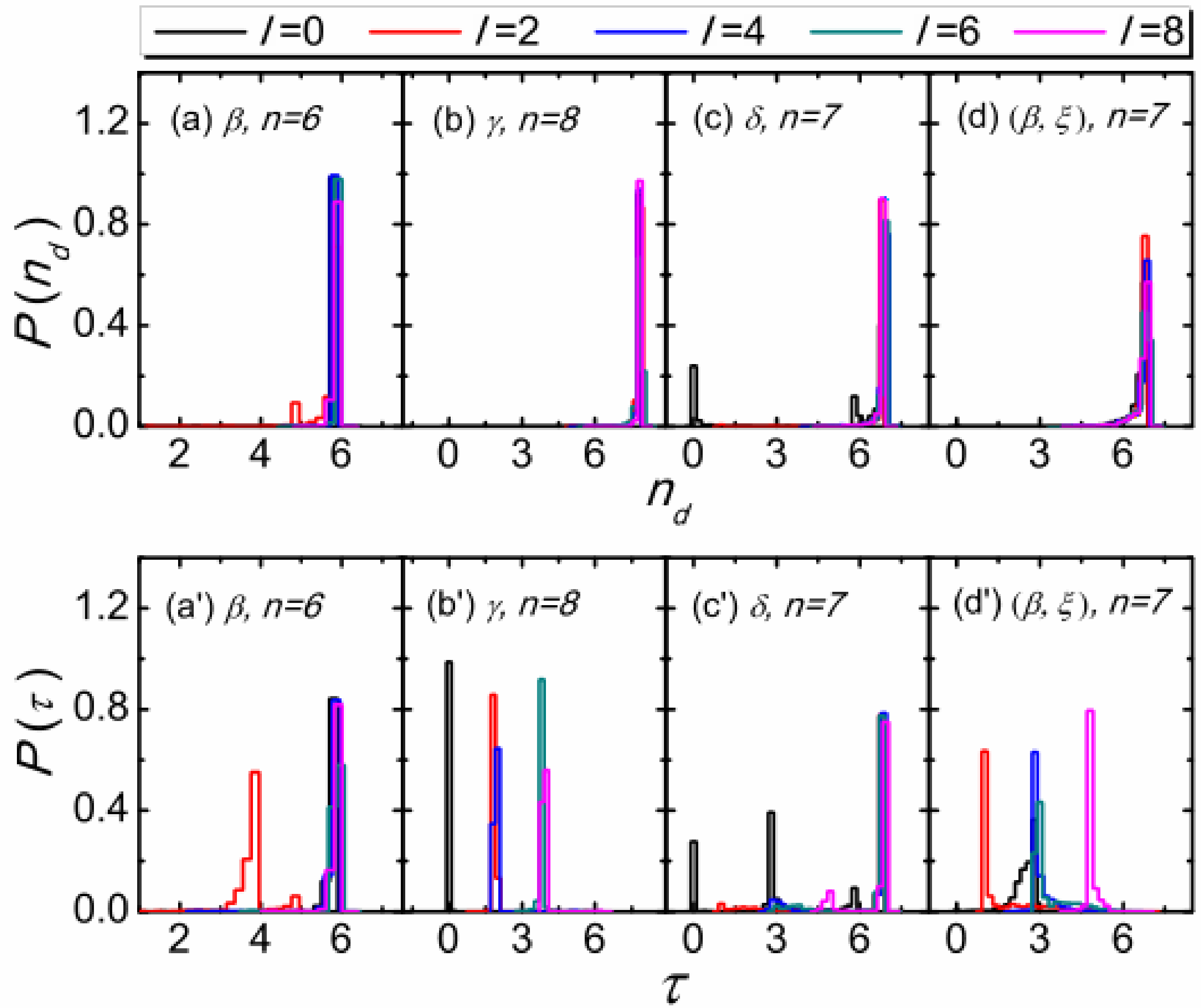
$$\beta : \ R_6 = \frac{21}{10}R_4,$$
$$\xi : \ R_8 = \frac{57}{14}R_4 - \frac{11}{7}.$$

Predictions given by the U(5) limit

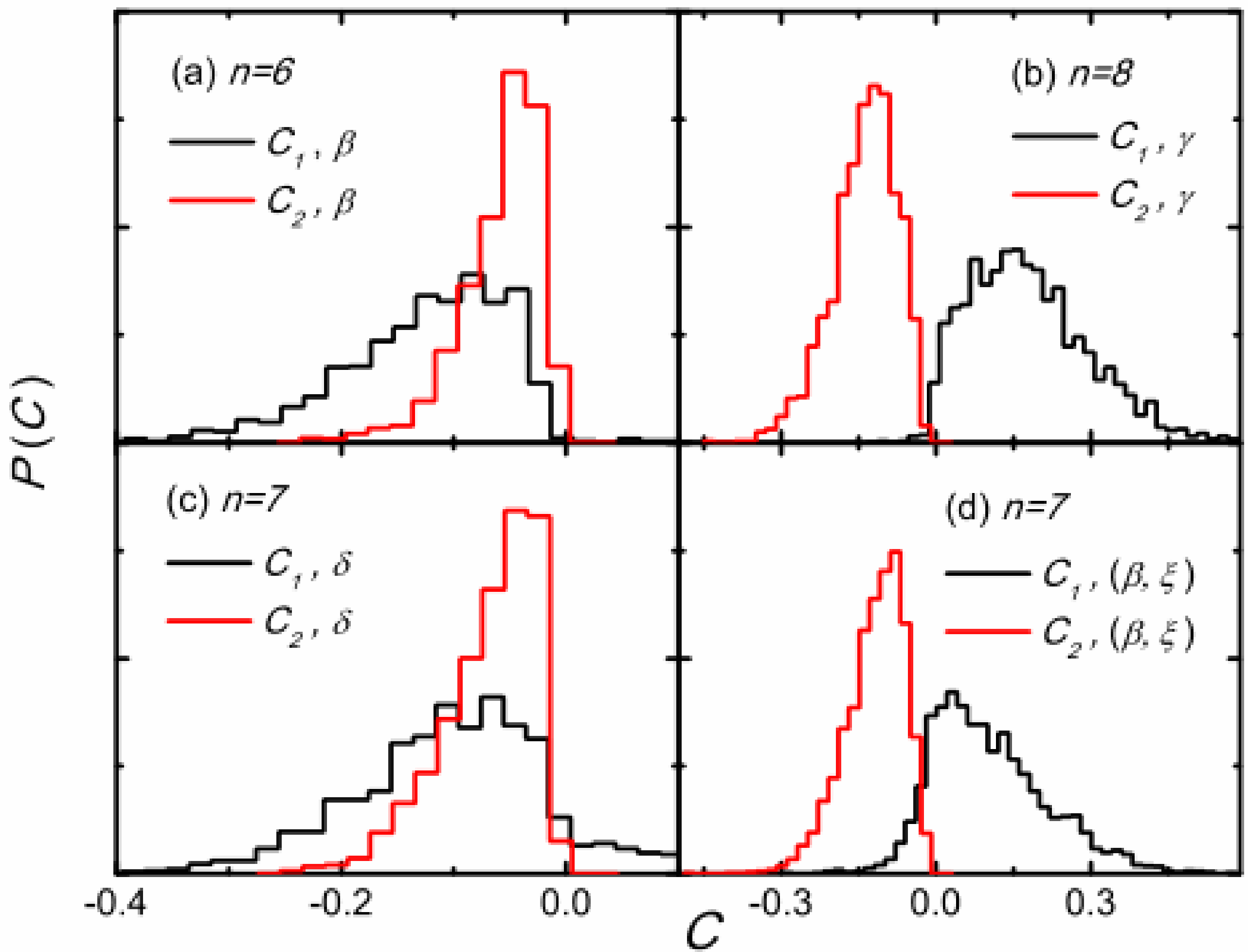
TABLE I: Summary of quantum numbers for yrast spin I states under different n, C_1 and C_2. n is the total number of sd bosons, and "g.s." is the abbreviation for "ground state". ▲ means more favored case, and △ means less favored case.

C_1, C_2	$C_1<0, C_2>0$						$C_1>0, C_2>0$			
n	$3k$		$3k+1$		$3k+2$		$2k$		$2k+1$	
τ,λ	τ	λ	τ	λ	τ	λ	τ	λ	τ	λ
0_1^+	n	0	$n-4$	0	$n-2$	0	0	0	3	0
2_1^+	$n-2$	1	n	1	n	2	2	2	1	1
4_1^+	n	3	n	4	n	2	2	2	3	3
6_1^+	n	3	n	4	n	5	4	4	3	3
8_1^+	n	6	n	4	n	5	4	4	5	5
g.s. spin	0		▲2, △ 0		▲2, △ 0		0		▲2, △ 0	
(R_4, R_6)	β		δ		δ		γ		β	
(R_4, R_8)	β		δ		δ		γ		ξ	
C_1, C_2	$C_1<0, C_2<0$						$C_1>0, C_2<0$			
n	$3k$		$3k+1$		$3k+2$		$2k$		$2k+1$	
τ,λ	τ	λ	τ	λ	τ	λ	τ	λ	τ	λ
0_1^+	n	0	$n-4$	0	$n-2$	0	0	0	3	0
2_1^+	$n-2$	1	n	1	n	2	2	2	1	1
4_1^+	n	3	n	4	n	2	2	2	3	3
6_1^+	n	3	n	4	n	5	4	4	3	3
8_1^+	n	6	n	4	n	5	4	4	5	5
$2n_1^+$	n	n	n	n	n	n	n	n	n	n
g.s. spin	$2n$		$2n$		$2n$		▲$2n$, △ *else*		▲$2n$, △ 2	
(R_4, R_6)	β		δ		δ		γ		β	
(R_4, R_8)	β		δ		δ		γ		ξ	

How good is the U(5) limit



How good is the U(5) limit



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Summary

- 1. With random interactions, the yrast states of the sd interacting boson model exhibit *highly compact* correlations.
- 2. These correlations well overlap with the predictions given by the U(5) limit.
- 3. d-boson condensation is a good approximation of most correlations
- Question: Why is d-boson condensation good, even under random interactions?

此情可待成追忆，

只是当时已惘然。

We still know so little about many-body quantum systems.

谢　谢！