Nucleon shape phase transition at the nucleon level within SD-pair shell mode

Yanan Luo(罗延安), Lei Li(李磊)

School of Physics, Nankai University, Tianjin

Yu Zhang(张宇), Feng Pan(潘峰)

Department of Physics, Liaoning Normal University, Dalian

J. P. Draayer

Department of Physics and Astronomy, LSU, BTR

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Outline

- **I. Nuclear Shapes and Shape Phase Transition**
- **II. Status of the Research on the nuclear shape Phase Transition**
- **III. SD-pair shell model(SDPSM)**
- **IV. Nuclear shapes in SD-pair shell model**
- **V. Nucear shape phase transition in SD-pair shell model**

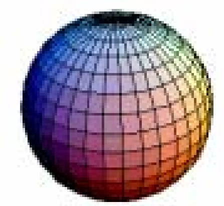
I. Nuclear Shape-Phases and Shape-Phase Transition

Modes of nuclear collective motion and the symmetries

♣ Shape of Nucleus

♠ Sphere

$$R = r_0 A^{1/3}$$



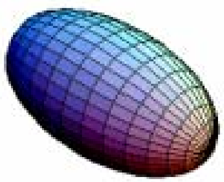
♠ Deforemation

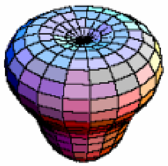
quadrupole

octupole

hexadecupole

$$R = R_0 [1 + \sum_{km} \alpha^*_{km} Y_{km}(\theta, \varphi)]$$





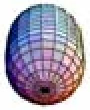
♣ Modes of Nuclear Collective Motion

vibration

axial rotation (prolate, oblate)

γ-soft rotation

triaxial rotation ●●● ●●● ●●● ●●●



From Prof. Yuxin Liu

♣ Correspondence between collective motion and symmetry

Vibration ⟷ U(5)

Axial Rotation ⟷ SU(3) (SU*(3))

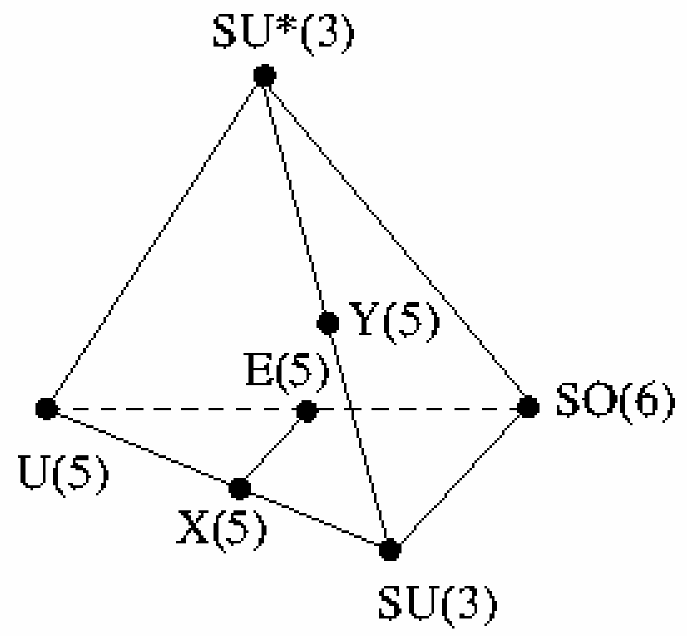
γ -soft rotation ⟷ O(6)

♠ Shape Phase Transition and the States at the Critical Points of the Phase Transitions

Vibration – γ -soft Rotation ⟶ E(5)

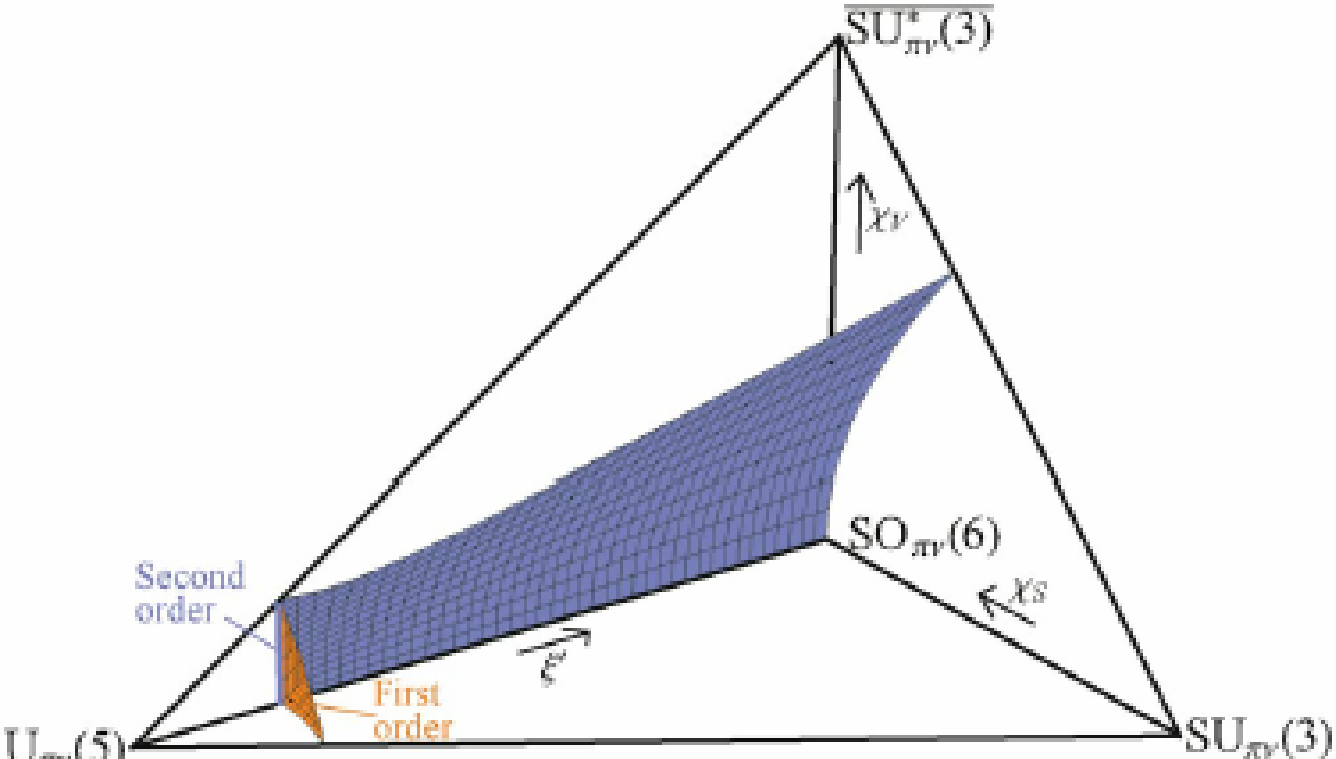
Vibration – Prolate Axial Rotation ⟶ X(5)

Prolate – Oblate Axial Rotations ⟶ Y(5)



Iachello, PRL 91, 132502 ('03)

Nuclear shape-phase and phase transition in IBM-2



M. A. Caprio and F. Iachello，Phys. Rev. Lett. **93,** 242502 (2004)

II. Status of the research on nuclear shape phase transition

Even-even nuclei, Odd-even nuclei, Odd-odd nuclei

1. IBM, IBFM, IBFFM

2. Geometric Collective models

3. Fermionic approachs
   FDSM, shell model, HFB

....

The most useful observables

- To explore the transitional patterns, the effective order parameters we used are:

$$v_2 = (< 0_2^+|\hat{n}_d|0_2^+ > - < 0_1^+|\hat{n}_d|0_1^+ >)/N$$
$$v_2' = (< 2_1^+|\hat{n}_d|2_1^+ > - < 0_1^+|\hat{n}_d|0_1^+ >)/N$$

F. Iachello and N. V. Zamfir, Phys. Rev. Lett. 92, 212501 (2004).

$$K_1 = B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$$
$$K_2 = B(E2; 0_2^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$$

Y. Zhang, et. al., Phys. Rev. C 76, 011305(R) (2007).

$$R_{42} = E_{4_1^+}/E_{2_1^+}$$
$$R_{60} = E_{6_1^+}/E_{0_2^+}$$

Dennis Bonatsos, et. al., Phys. Rev. Lett. 100, 142501 (2008)

- Two neutron seperation energies
- E0 transition
- Isotopes shifts
- Two neutron transfer cross sections
- …………………

III. SD-pair shell model(SDPSM)

The Hamiltonian used in the SDPSM is

$$H = \sum_{\sigma=\pi,\nu} (H_0(\sigma) + V(\sigma)) + \sum_t \kappa_t Q_\pi^t \cdot Q_\nu^t,$$

$$H_0 = \sum_a \epsilon_a \hat{n}_a, \quad V = \sum_s g_s A^{s\dagger} \cdot A^s + \sum_t k_t Q^t \cdot Q^t,$$

$$A_\nu^{r\dagger} = \sum_{cd} y(cdr)(C_c^\dagger \times C_d^\dagger)_\nu^r,$$

Because of the success of the IBM, the full space was truncated into SD-pair subspace, r=0, 2

$$A_{M_N}^{J_N\dagger} = \left(\cdots \left(\left(A^{r_1\dagger} \times A^{r_2\dagger} \right)^{J_2} \times A^{r_3\dagger} \right)^{J_3} \times \cdots \times A^{r_N\dagger} \right)_{M_N}^{J_N}$$

The Hamiltonian can be diagonolized directly in the Fermion space!

IV. Nuclear shape phases in SDPSM

- The Hamiltonian used here is

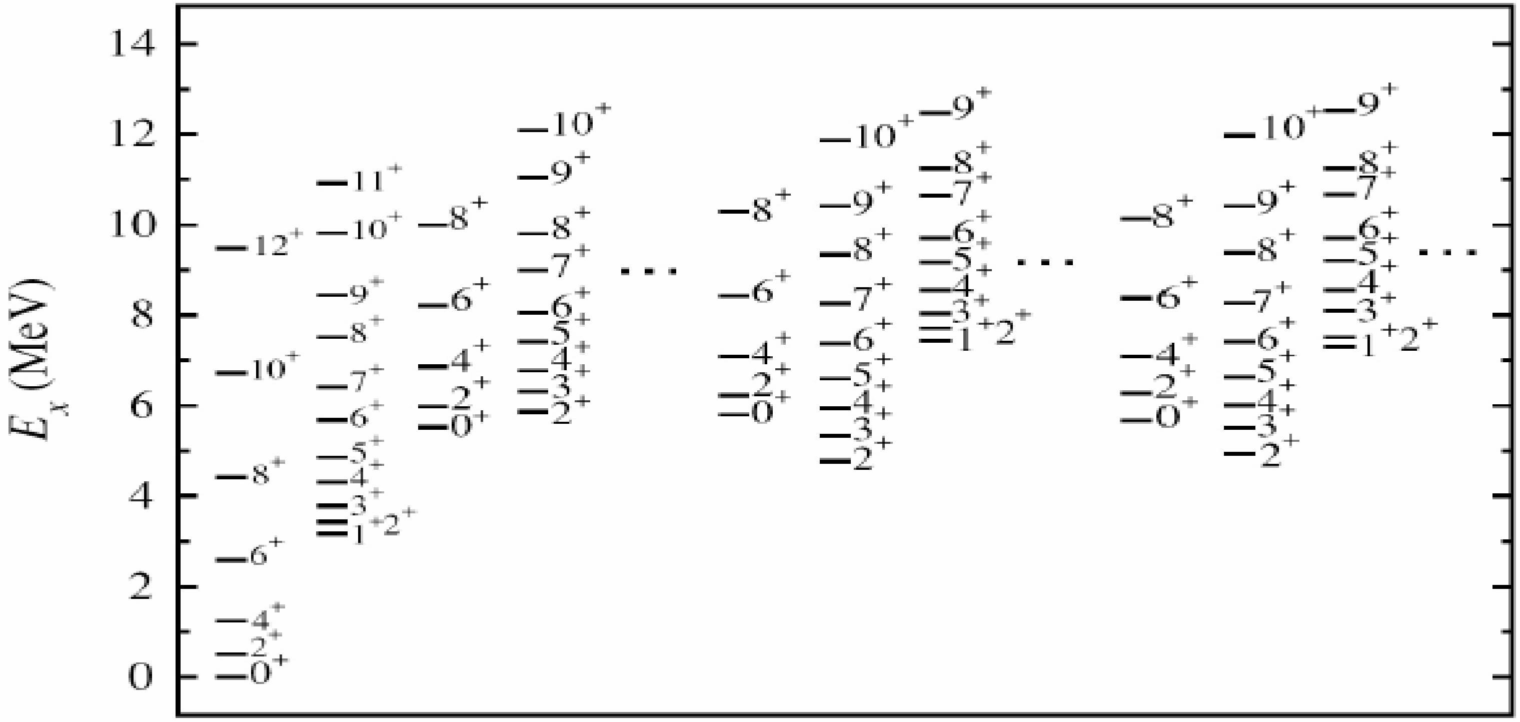
$$H = \sum_{\sigma=\pi,\nu} (H_0(\sigma) + V(\sigma)) + \kappa_2 Q^2_\pi \cdot Q^2_\nu,$$

$$H_0 = \sum_a \epsilon_a \hat{n}_a, \quad V = g_0 A^{0\dagger} \cdot A^0 + k_2 Q^2 \cdot Q^2,$$

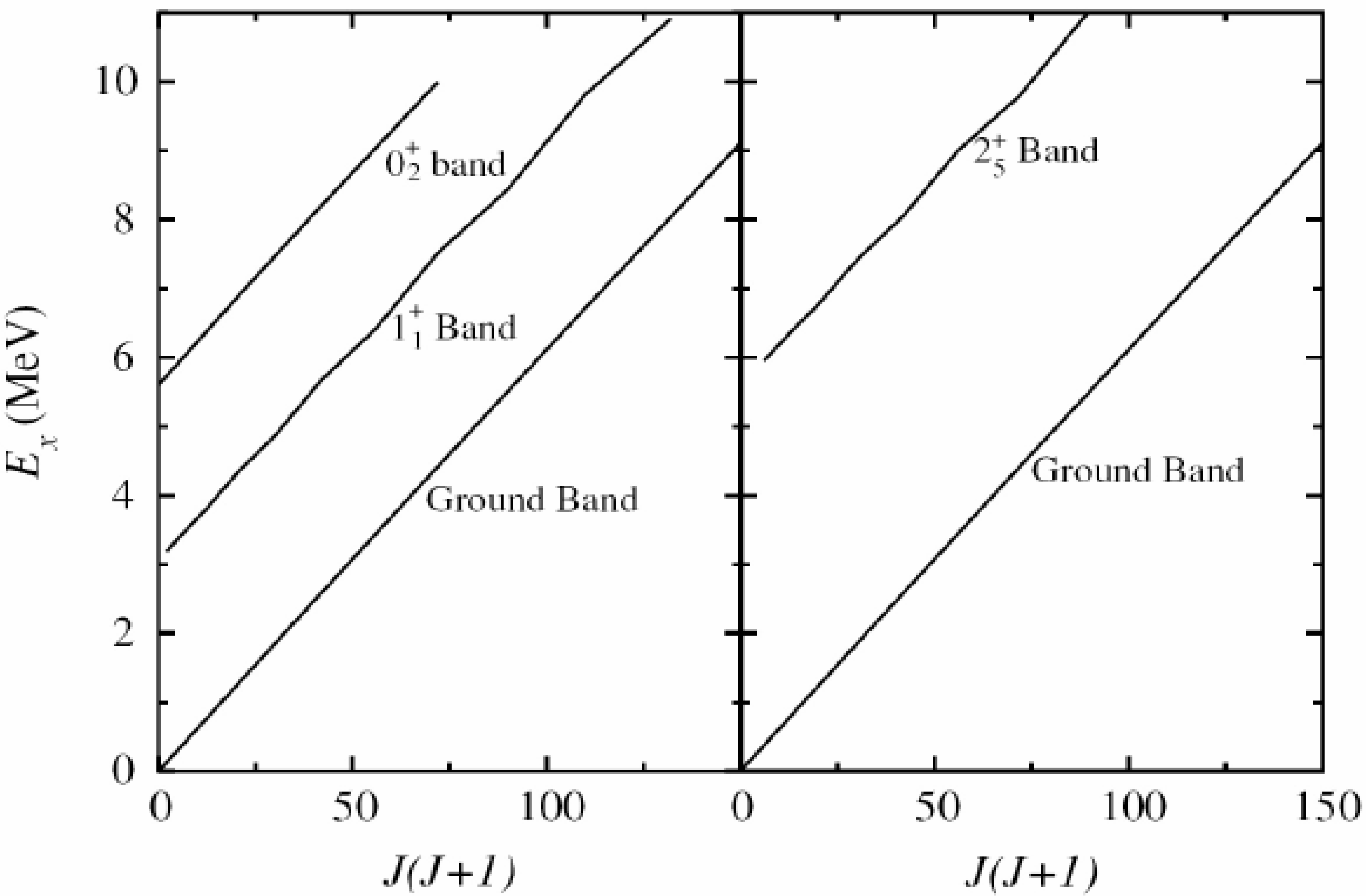
$$Q^t = \sum_{i=1}^{n} (r_i)^t Y_t(\theta_i \phi_i),$$

Y. Luo, F. Pan, et. al., Phys. Rev. C *71(2005)044304*
Y. Luo, F. Pan., et. al., Chin. Phys. Lett. *22(2005)366*
L. Li, et. al., J. Phys. G: nucl. & part. *36(2009)125107*
L. Li, et. al., Int. J. Mod. Phys. E *36(2011)125107*

Rotational spectrum



- A pure quadrupole-quadrupole interaction with $\kappa_\pi = \kappa_\nu = 1/2\kappa$ was used for $N_\pi = N_\nu = 3$ system.



E_x (MeV)
10
8
6
4
2
0
0_2^+ band
1_1^+ Band
Ground Band
2_5^+ Band
Ground Band
0
50
100
150
J(J+1)
J(J+1)

V. Nuclear shape phase transition in SDPSM

- To study the phase transitional patterns, the Hamiltonian for proton-neutron coupled system is written as

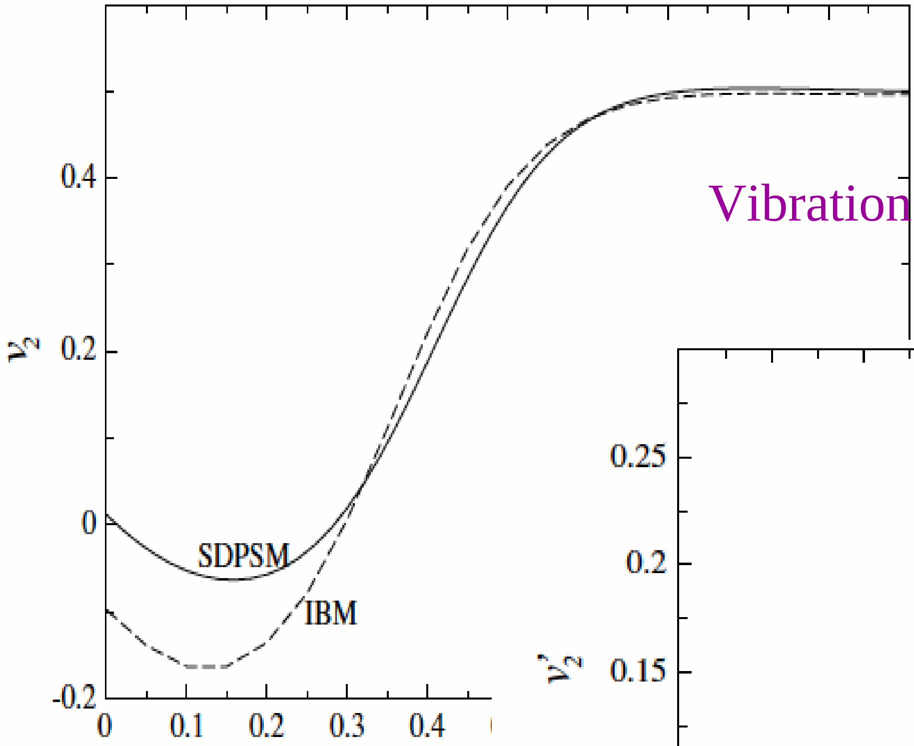
$$H = (1-\alpha)H_{U(5)} + \alpha H_X, \qquad (15)$$

where $0 \leq \alpha \leq 1$ is a control parameter,

$$\begin{array}{lll} H_X \longrightarrow H_{SU(3)} & vibration-rotation\ transitional\ patterns \\ H_X \longrightarrow H_{SO(6)} & vibration-\gamma-soft\ transitional\ patterns \end{array}$$

Y. Luo, F. Pan, et. al., Phys. Rev. C *73(2006)044323;*
Y. Luo, Y. Zhang, et. al., Phys. Rev. C*80(2009)014311*
F. Y. Wang, L. Liu, et. al., Chin. Phys. Lett. *25(2008)2432*

Vibration to rotational transitional pattern



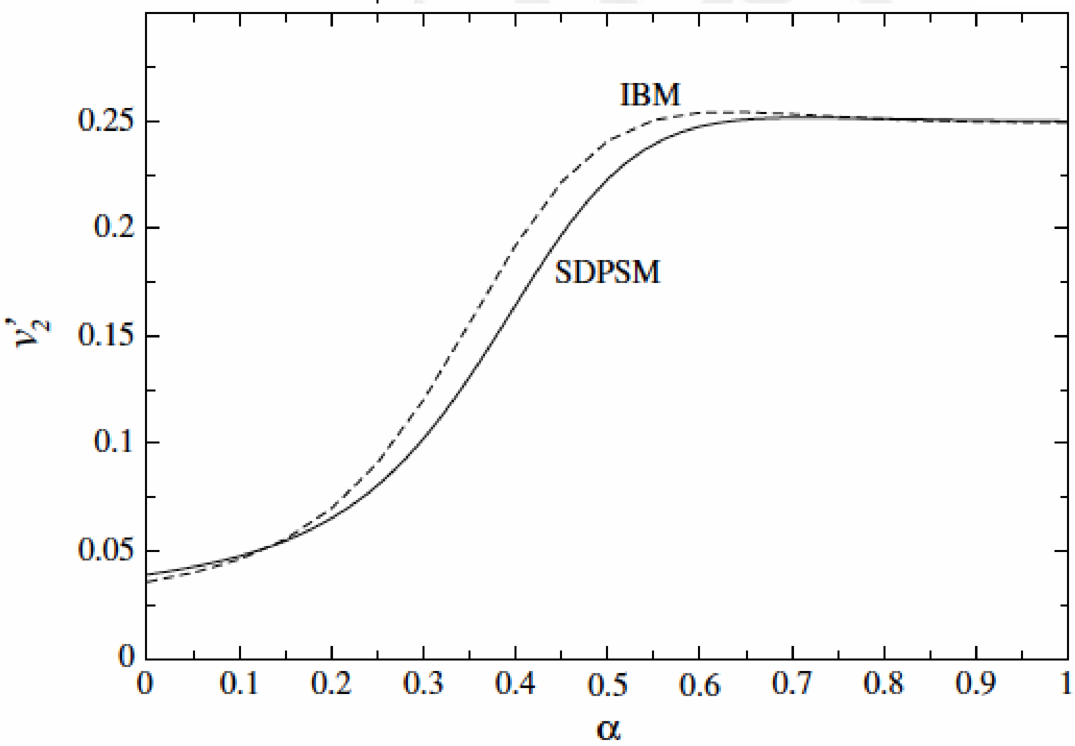
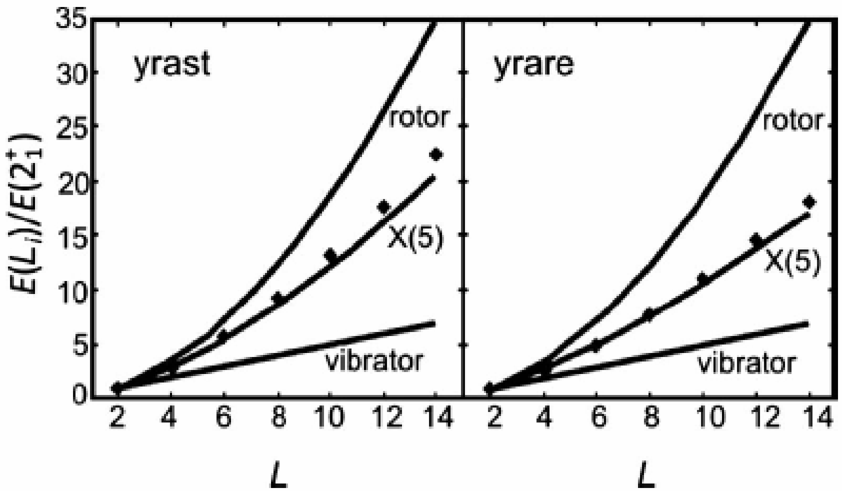


TABLE II: Energy and B(E2) ratios at vibrational, rotational limit, and $X(5)$-like critical point calculated in the SDPSM.

limit	$\frac{E_{4_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{0_2^+}}$	$\frac{4_1^+\to 2_1^+}{2_1^+\to 0_1^+}$	$\frac{6_1^+\to 4_1^+}{2_1^+\to 0_1^+}$
vibrational limit	1.99	2.97	1.47	1.49	1.48
$X(5)$-like point	2.91	5.60	1.05	1.38	1.38
rotational limit	3.33	6.96	0.46	1.34	1.32
	$\frac{E_{0_2^+}}{E_{2_1^+}}$	$\frac{E_{2^+}-E_{0_2^+}}{E_{2_1^+}}$	$\frac{E_{4^+}-E_{0_2^+}}{E_{2_1^+}}$	$\frac{2^+\to 0_2^+}{2_1^+\to 0_1^+}$	$\frac{4^+\to 2^+}{2_1^+\to 0_1^+}$
$X(5)$-like point (0_2^+ band)	5.32	2.30	5.33	0.37	0.43



The IBM results

$$R_{42} = 2.91$$
$$R_{60} = 1.0$$
$$E_{0_2}/E_{2_1} = 5.67$$

Liu YX, et. al., Phys. Lett. B688,298(2010)

$$H_F = \sum_{jm} E_j a^{\dagger}_{jm} a_{jm} - \frac{1}{2} g_0 \hat{P}^{\dagger}_0 \hat{P}_0 - \frac{1}{2} g_2 \sum_{\mu} \hat{P}^{\dagger}_{2\mu} \hat{P}_{2\mu} - \frac{1}{2} k \hat{Q}_{2\mu} \cdot \hat{Q}_{2\mu}$$

↓ Boson Mapping (Dyson)

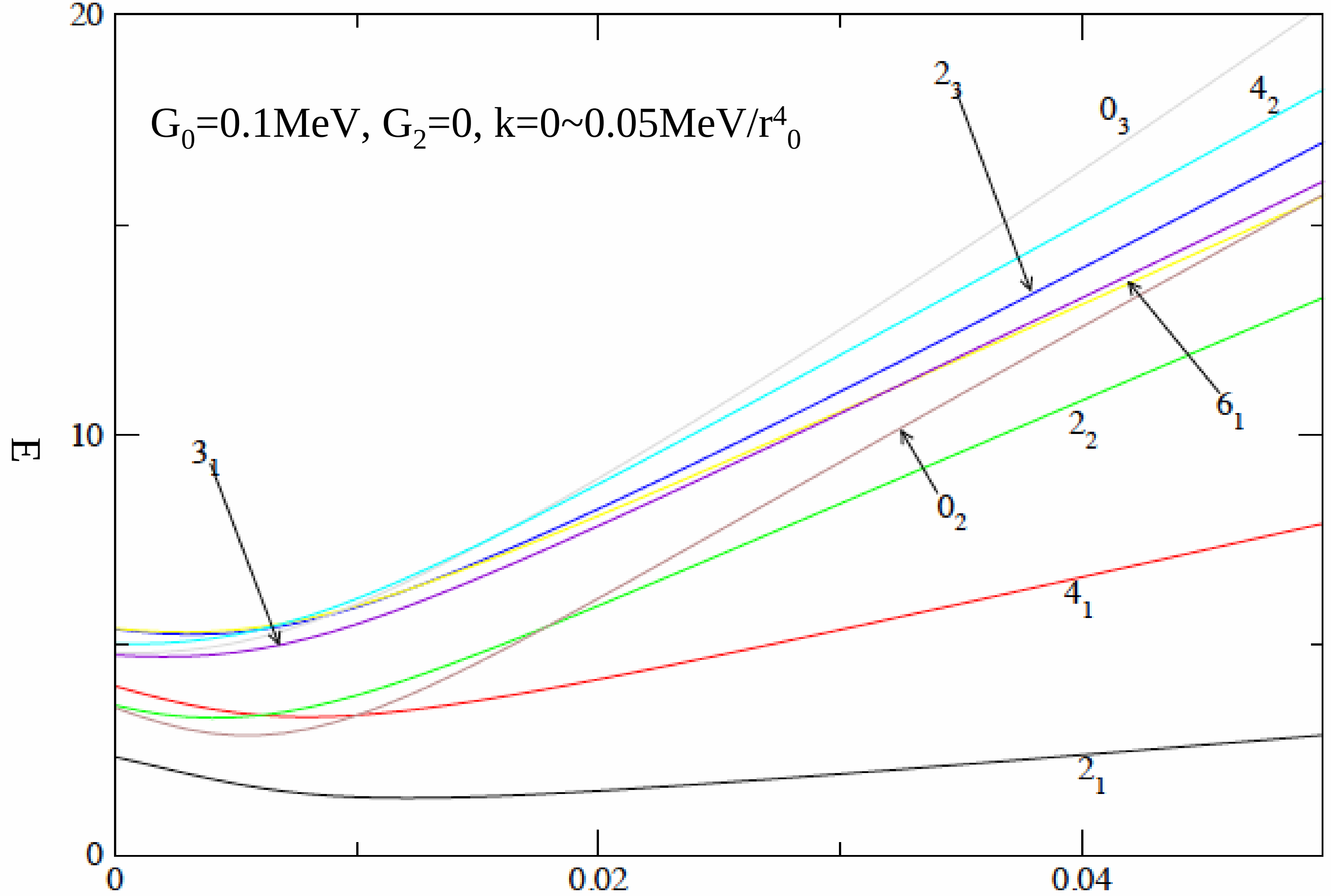
$$H_B(s^{\dagger}, d^{\dagger})$$

The Hamiltonian used in the SD-pair shell model

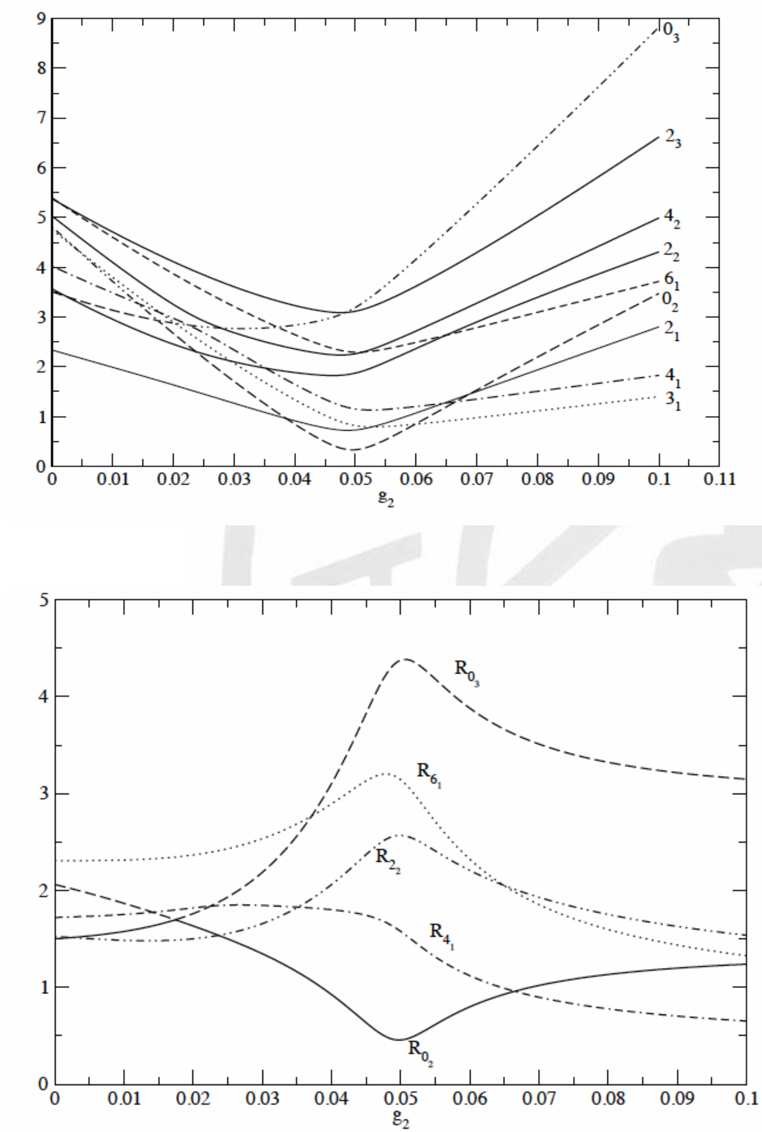
$$H = H_0 - G_0 \mathcal{S}^\dagger \mathcal{S} - G_2 \mathcal{P}^\dagger \mathcal{P} - \kappa Q^{(2)} \cdot Q^{(2)}$$

50-82 shell $j = 1/2,\ 3/2,\ 5/2,\ 2/7$ and $11/2$.

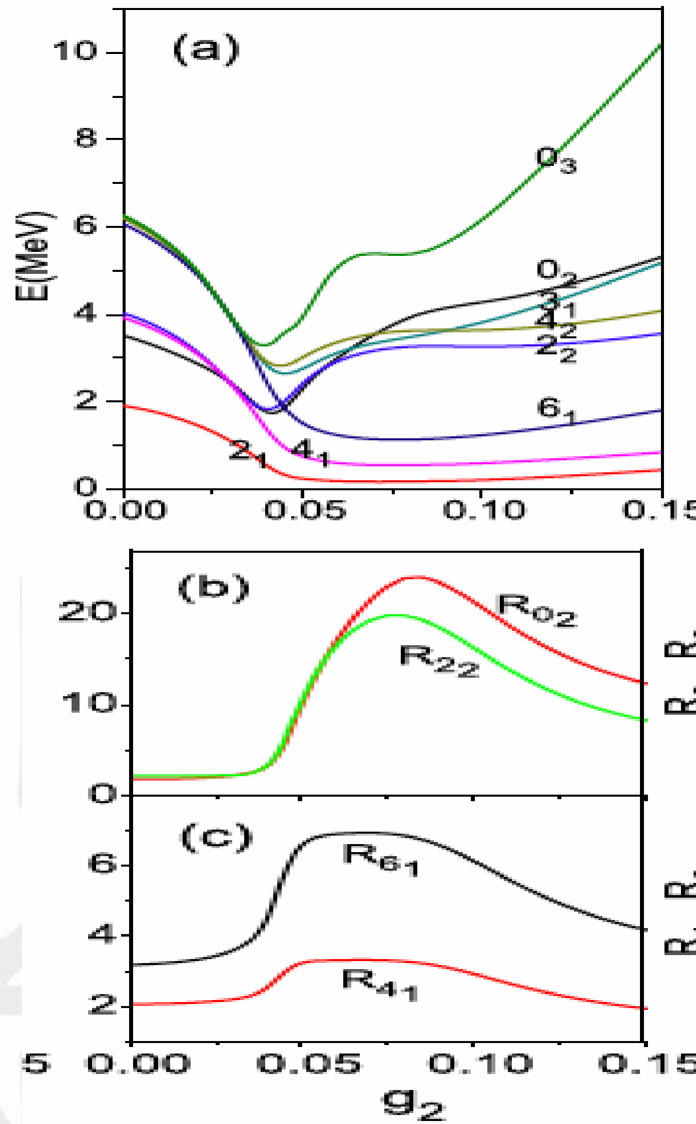
1.3, 2.8, 0, 0.8 and 2.5



Quadrupolel-quadrupole interactional strength kappa



SDPSM results



DBM results

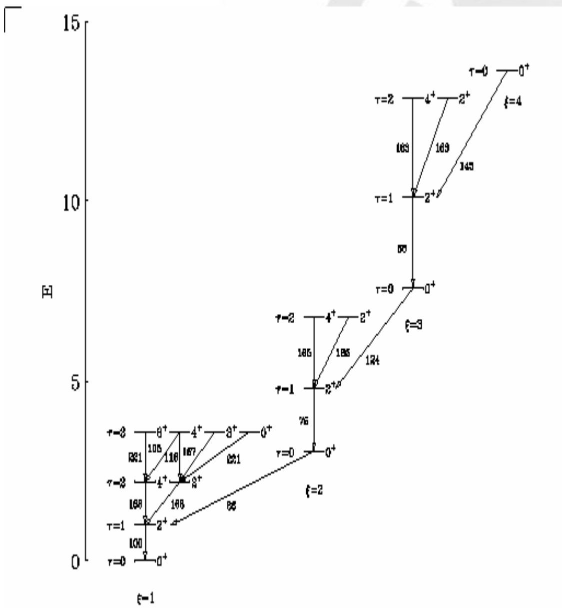
Summary

- The nuclear shape phases can be produced very well in the SDPSM
- The nulcear shape phase transitional pattern as in the IBM can be produced in the SDPSM
- The properties of the critical symmetry can be produced in the SDPSM

Thanks !

Spectrum of E(5) Symmetry | Spectrum of X(5) Symmetry | Spectrum of Y(5) Symmetry

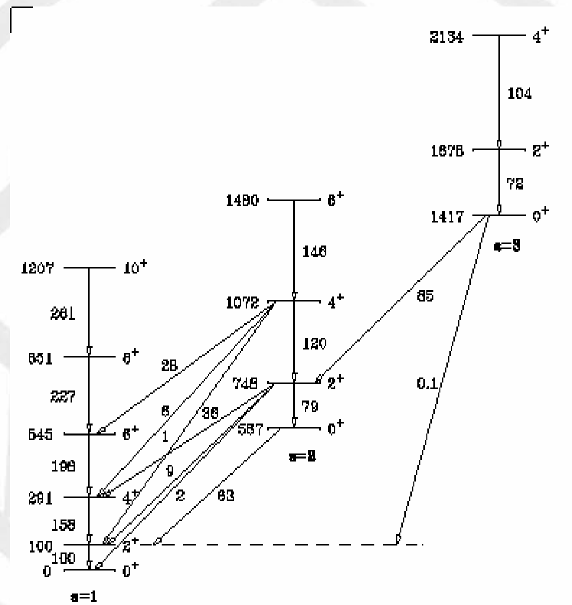
$$E(n_\beta, L, s', K, M) = E_0 + B' n_\beta + BL(L+1) + A(x_{s',M'})^2,$$



(Iachello, PRL 85, 3580 (2000))

^{134}Ba, ^{108}Pd, ^{130}Xe, ···

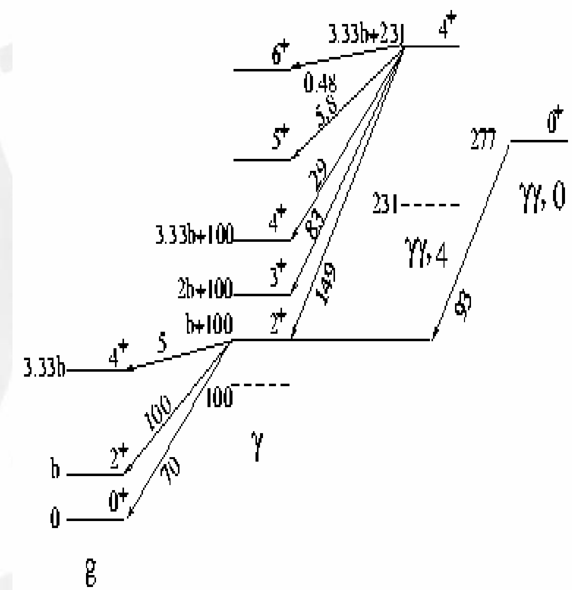
(Casten, PRL 85, 3584 (2000); Ginocchio, PRL 90, 212501 (2003) ; Liu, PRC 65, 057301 (2002), …)



(Iachello, PRL 87, 052501(2001))

^{152}Sm, ^{154}Gd, ^{156}Dy, ^{150}Nd, ···

(Casten, PRL 87, 052503 (2001); Capirio, PRC 66, 054310 (2002); Tonev, PRC 69, 034334 (2004); …)

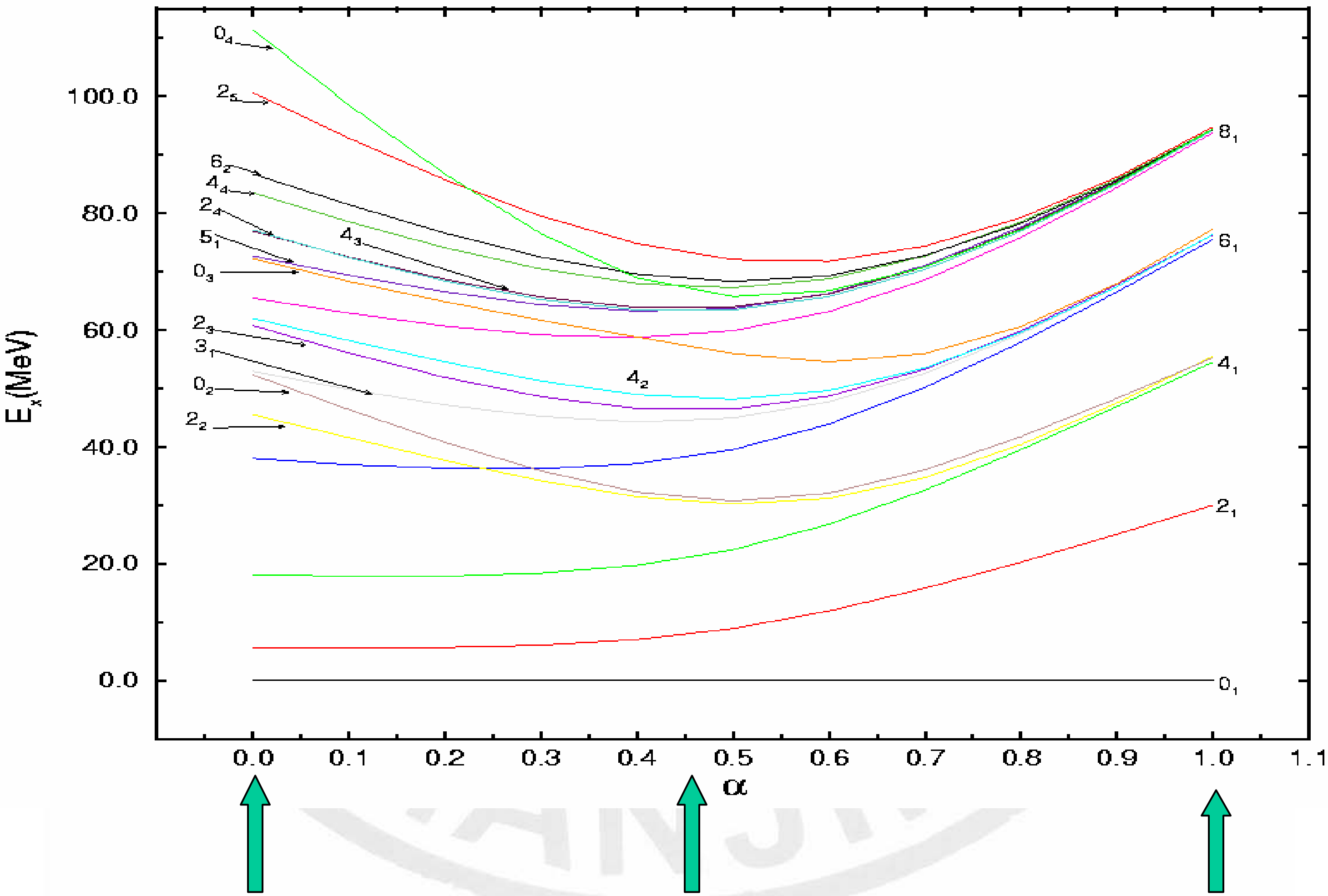


(Iachello, PRL 91, 132502(2003))

^{166}Er, ^{168}Er, ···

(PRC 68 , 024307 (2003); …

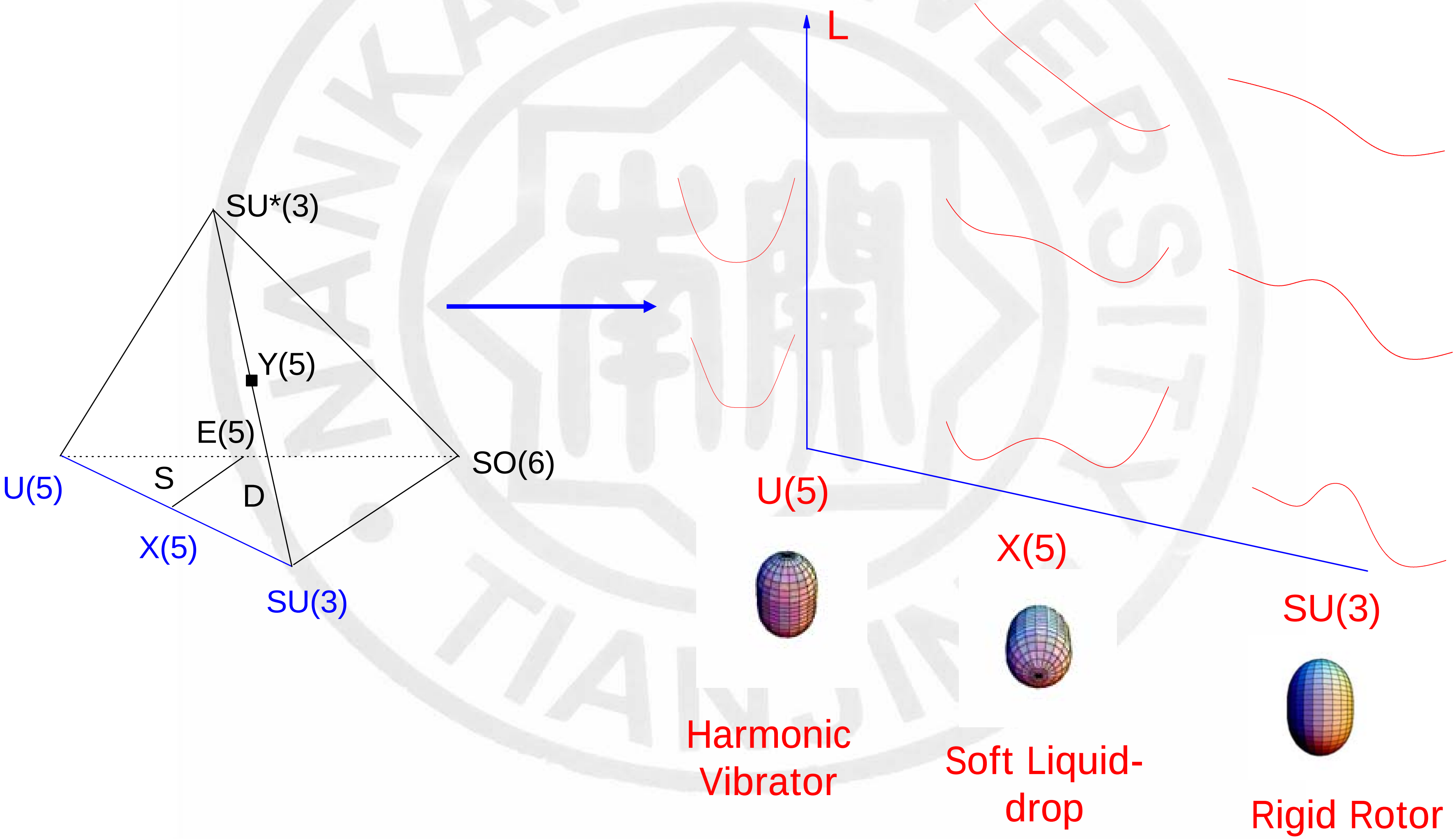
Characteristic of Evolution of Energy Spectrum for the Transition from U(5) to SU(3) Through X(5)



SU(3) (Rotation) X(5) U(5) (Vibration)

(Pan, Draayer, et. al., PLB 576, 297 (2003))

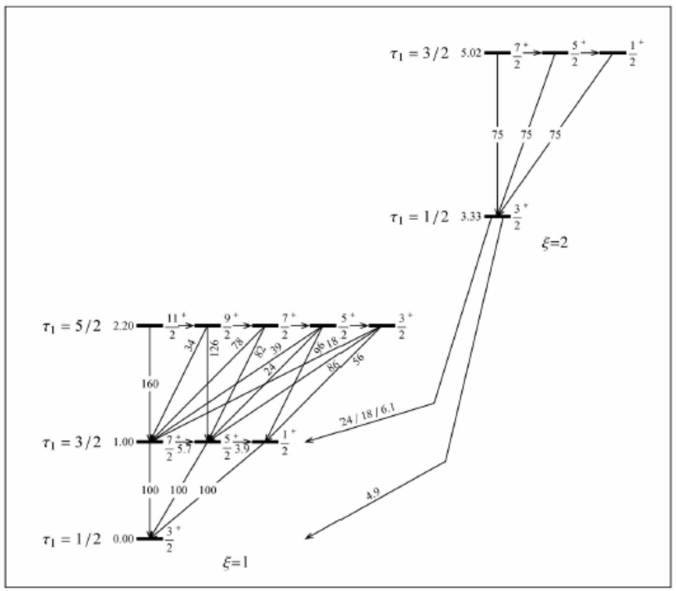
Summary for the shape-phase transition in the U(5)-SU(3) transitional region

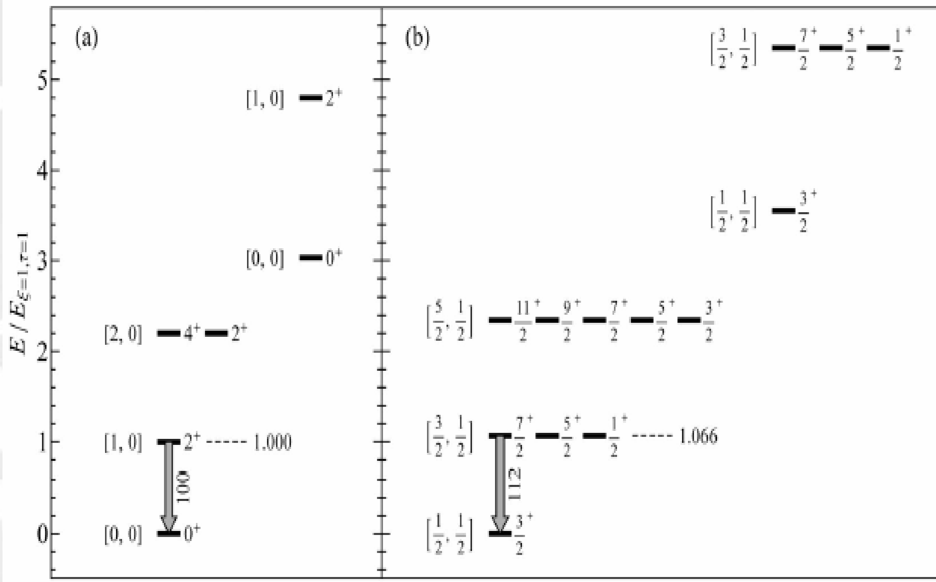


Extension:

Unified description of the shape phase structure and phase transition of odd-A nuclei

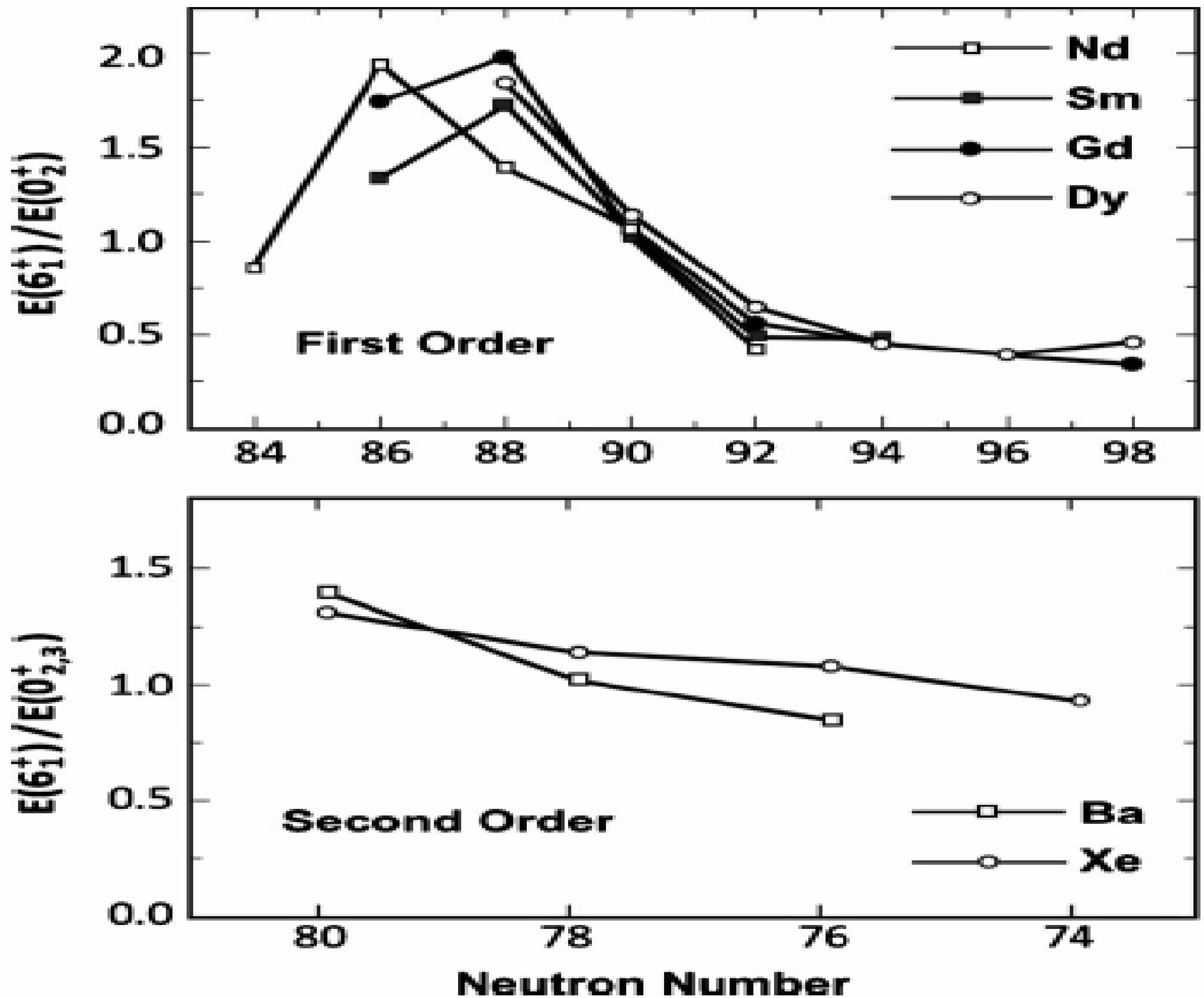
F. Iachello, Phys. Rev. Lett. 95, 052503 (2005);





Result of U(6,4) model

^{135}Ba, E(5/4) Symmetry, M.S. Fetea, et al., Phys. Rev. C 73, 051301 (2005)


Nd
Sm
Gd
Dy
First Order
E(6+1)/E(0+2)
2.0
1.5
1.0
0.5
0.0
84
86
88
90
92
94
96
98
Ba
Xe
Second Order
E(6+1)/E(0+2,3)
1.5
1.0
0.5
0.0
80
78
76
74
Neutron Number

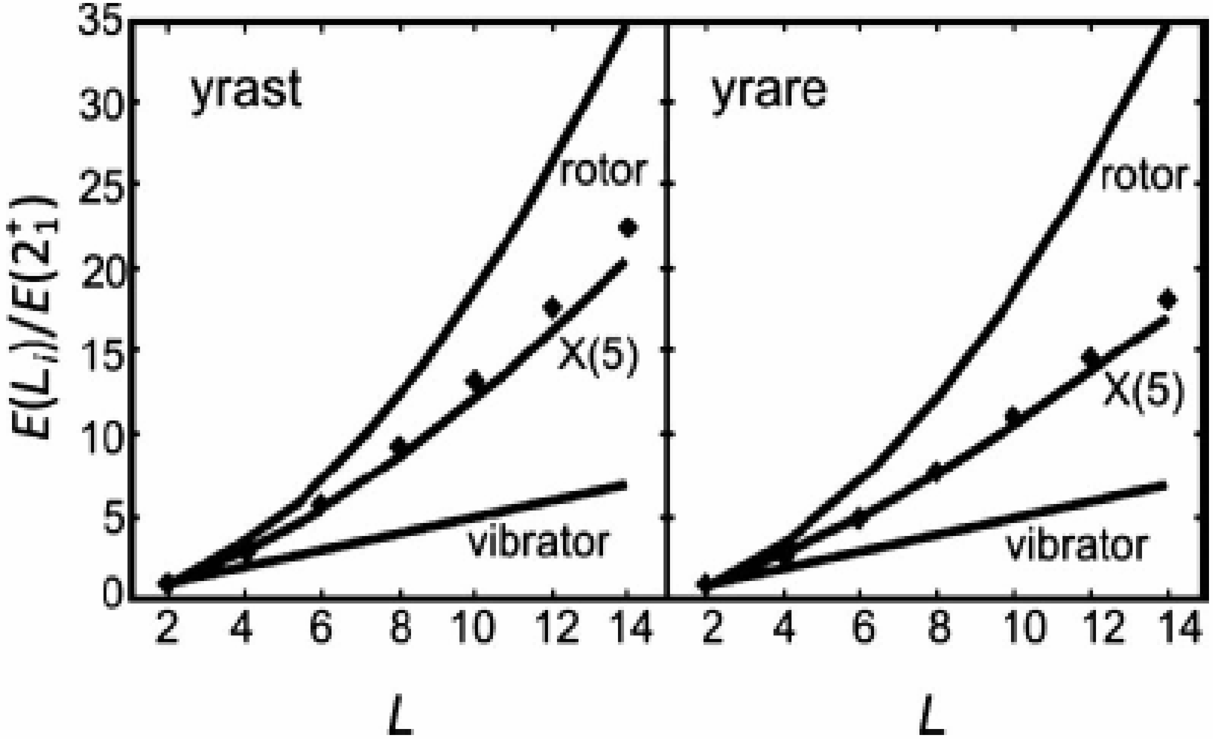
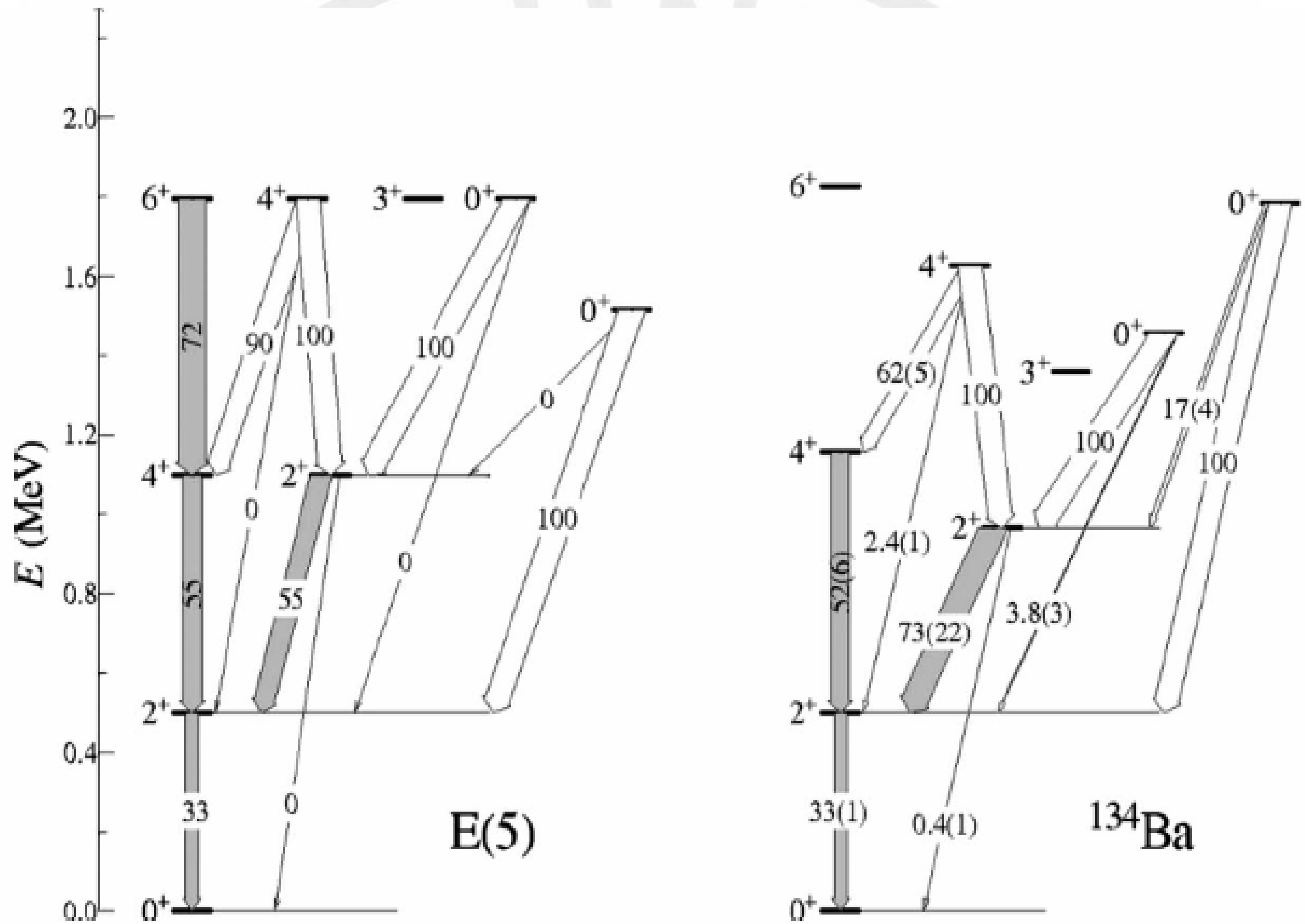
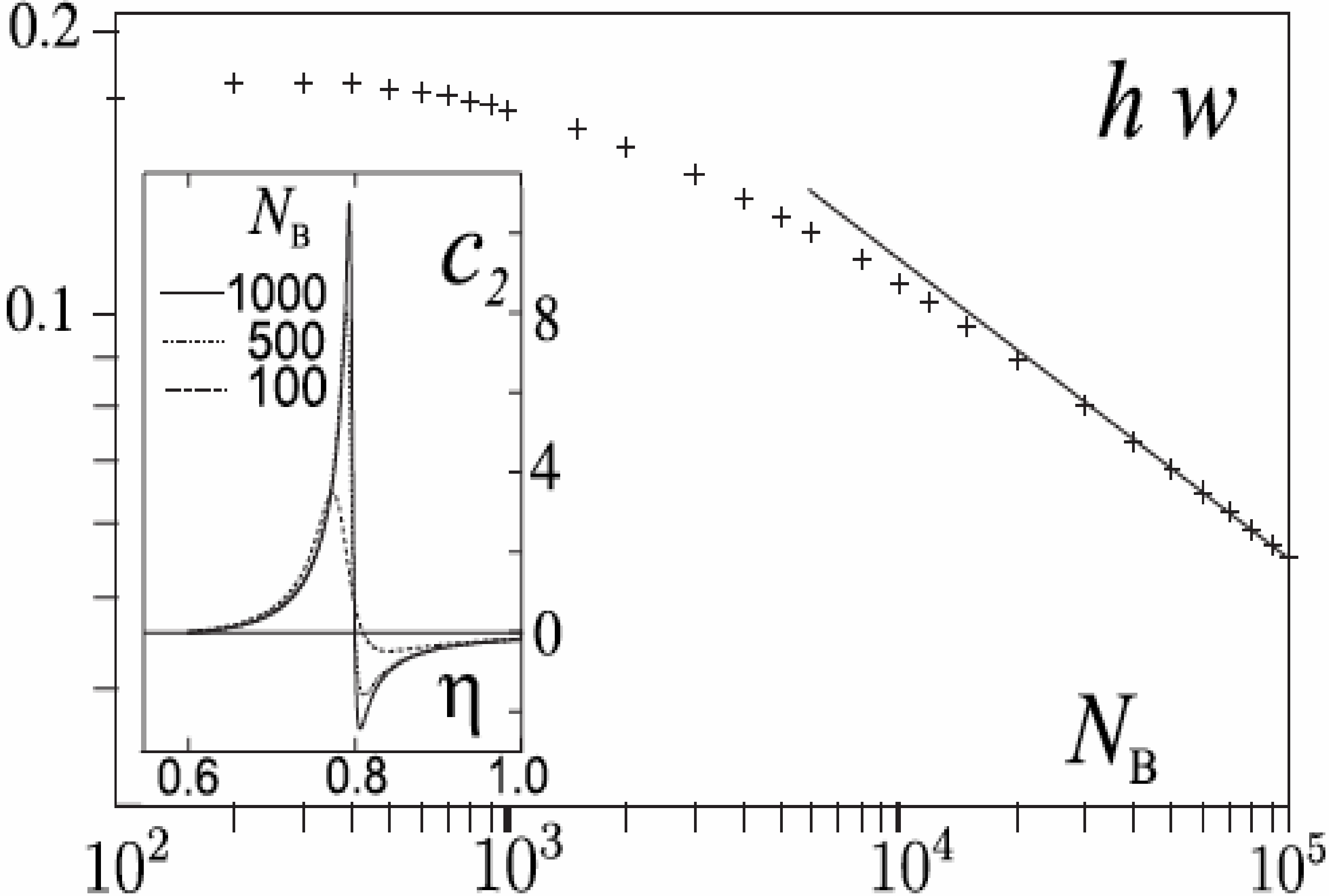


FIG. 32. Values of $E(L)/E(2_1^+)$ for yrast and yrare levels in ^{152}Sm compared to the harmonic vibrator, symmetric rotor, and X(5). Adapted from Casten and Zamfir, 2001.



E (MeV)
2.0
1.6
1.2
0.8
0.4
0.0
6+
4+
3+
0+
0+
4+
2+
2+
0+
72
90
100
100
0
0
100
0
55
55
33
0
E(5)
6+
0+
4+
0+
3+
62(5)
100
17(4)
100
100
4+
2.4(1)
2+
52(6)
73(22)
3.8(3)
2+
33(1)
0.4(1)
0+
134Ba



0.2
0.1
h w
N_B
10^2
10^3
10^4
10^5
N_B
1000
500
100
c_2
8
4
0
η
0.6
0.8
1.0