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Symmetry Restoration & Quantumness Reestablishment

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Outline

- Introduction
- Theory and Model
  - BCS & PBCS
  - Entanglement & Quantum Discord
- Results & Analysis
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  - Applications to Realistic Nuclei
- Conclusions & Outlook

I. Introduction

Quantum many-body systems are usually characterized by a microscopic Hamiltonian → Approximation Methods

Mean Field Theory——simple, effective, powerful
At a price of breaking the symmetry of the Hamiltonian of the system, e.g., BCS method——breaking the conservation of particle number

Fluctuation of particle number can be neglected for the bulk superconductors, etc., but not for systems with limited particle number, such as superconducting grain, nuclear systems.

A. Mastellone, G. Falci and R. Fazio, Phys. Rev. Lett. **80**, 4542 (1998).

I. Introduction

The breaking of Hamiltonian symmetry will wash out the quantumness (量子性) of the system.

Quantumness is described by entanglement (纠缠) and quantum discord (量子失谐) between different modes

Concurrence: a measure of entanglement
Quantum discord: beyond entanglement

Motivation: exploiting projected BCS (PBCS) to restore the system’s symmetry (conversation of particle number); reestablish the quantumness, investigate the quantumness vs particle number and interaction strength.

I. Introduction

G.-M. Zeng, L.-A. Wu & H.-J. Xing, Scientific Reports, **4**, 6377 (2014).

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Symmetry restoration and quantumness reestablishment

SUBJECT AREAS:
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QUBITS

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A realistic quantum many-body system, characterized by a generic microscopic Hamiltonian, is accessible only through approximation methods. The mean field theories, as the simplest practices of approximation methods, commonly serve as a powerful tool, but unfortunately often violate the symmetry of the Hamiltonian. The conventional BCS theory, as an excellent mean field approach, violates the particle number conservation and completely erases quantumness characterized by concurrence and quantum discord between different modes. We restore the symmetry by using the projected BCS theory and the exact numerical solution and find that the lost quantumness is synchronously reestablished. We show that while entanglement remains unchanged with the particle numbers, quantum discord behaves as an extensive quantity with respect to the system size. Surprisingly, discord is hardly dependent on the interaction strengths. The new feature of discord offers promising applications in modern quantum technologies.

2、Theory & Model —— BCS & PBCS

BCS was proposed at first to deal with superconductors and later generalized to the low lying states of nuclei which also have strong pair correlations.

1. J. Bardeen, L. N. Cooper & R. S. Schriffer, Phys. Rev. **108**, 1175 (1957).
2. B. F. Bayman, Nucl. Phys. **15**, 33 (1960).

We will concentrate on discussing the applications of BCS on nuclear systems and expect the results can be generalized to any limited correlated systems.

$$\left|\mathrm{BCS}\right\rangle = \prod_k \left(u_k + v_k a_k^\dagger a_{\bar{k}}^\dagger\right)\left|0\right\rangle, \quad u_k^2 + v_k^2 = 1$$

$$\left\langle\mathrm{BCS}\right|\hat{N}\left|\mathrm{BCS}\right\rangle = N,\ \delta E = 0 \rightarrow \text{parameters}, \left|\mathrm{BCS}\right\rangle,\ E_{\mathrm{g.s.}}$$

2、Theory & Model —— BCS & PBCS

1. Not conservative for particle number;
2. Separable (not entangled): lost of quantumness

The symmetry can be restored by making use of projective technology. How about the quantumness?

1. P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
2. K. Ditrich, H. J. Mang and J. H. Pradal, Phys. Rev. **135**, B22 (1964).

$$\hat{P}^{A} = \frac{1}{2\pi i}\oint \frac{z^{\hat{N}}}{z^{A+1}}dz$$

$$\left|\Psi^{N}\right\rangle = \hat{P}^{N=2p}\left|\Phi\right\rangle = \frac{1}{2\pi i}\oint \frac{d\zeta}{\zeta^{p+1}}\prod_{k>0}\left(u_k + v_k\zeta a_k^{?} a_{\bar{k}}\right)\left|0\right\rangle$$

2、Theory & Model —— Paring Force

Paring correlations can be well described by Paring Force Hamiltonian: Simple form, comprehensive applications

$$H = \sum_{k>0} \varepsilon_k \left(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}} \right) - \sum_{k,k'>0} G_{kk'kk'} a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}'} a_{k'}$$

Construct the following bilinear operators

$$S_+^{(k)} = a_k^\dagger a_{\bar{k}}^\dagger ;\ S_-^{(k)} = a_{\bar{k}} a_k ;\ S_0^{(k)} = \tfrac{1}{2}\left(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}} - 1 \right)$$

$$\mathbf{S}^{(k)} \equiv \left\{ S_0^{(k)}, S_+^{(k)}, S_-^{(k)} \right\} \to \text{behave as spin operators, } s = \tfrac{1}{2}$$

$$S_0^{(k)} \text{ eigenvalues:} \pm \tfrac{1}{2},\ \left| k\bar{k} \right\rangle \text{ occupied or not} \to \pm$$

$$\text{unoccupied} \to |0\rangle,\ \text{occupied} \to |1\rangle \ \Rightarrow\ \text{qubit}$$

2、**Theory & Model** —— Concurrence

Entanglement: a resource of quantum computation and quantum information.
Concurrence: a measure of bipartite entanglement （并发度）

W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

For two-qubit state $\rho_{AB} \equiv \rho$, introduce

$$\tilde{\rho} = \left(\sigma_y \otimes \sigma_y\right) \rho^* \left(\sigma_y \otimes \sigma_y\right)$$

$$\rightarrow \mathbb{C}(\rho) = \max\left(0,\ \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

λ_i : eigenvalues of $\rho\tilde{\rho}$ in descending order

when $\mathbb{C} > 0$, ρ is entangled

2、Theory & Model —— Beyond Entanglement

Entanglement does not cover all quantum correlations in a quantum systems.

Among other measures, **quantum discord** is one of the most popular one.

Even a separable state may have non-vanishing discord.
➔ Discord is more essential.

1. H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
2. A. Datta, A. Shaji and C. M. Caves, Phys. Rev. Lett. **100**, 050502 (2008).
3. ……

2、Theory & Model —— Quantum Discord

For a state described by $\rho_{AB} \equiv \rho$, **quantum discord**

$$\mathcal{D}(\rho) := \mathcal{I}(\rho) - \mathcal{C}(\rho)$$

where

$$\mathcal{I}(\rho) = \mathcal{S}(\rho_A) + \mathcal{S}(\rho_B) - \mathcal{S}(\rho)$$

$\rightarrow$ quantum analogue of classical **mutual information**

$\mathcal{S}(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ $\rightarrow$ **von Neumann entropy**

$$\mathcal{C}(\rho) = \mathcal{S}(\rho_A) - \min_{\{B_i\}} \mathcal{S}(\rho_X | \{B_i\})$$

$\rightarrow$ **Classical correlation**

$$\mathcal{S}(\rho_X | \{B_i\}) = p_0 \mathcal{S}(\rho_0) + p_1 \mathcal{S}(\rho_1)$$

$\rightarrow$ **conditional entropy**

2、Theory & Model —— Quantum Discord

Quatum Discord for two-qubit system

1. S. Luo, *Quantum discord for two-qubit systems*. Phys. Rev. A **77**, 042303 (2008).
2. M. Ali, A. R. P. Rau and G. Alber, Phys. Rev. A **81**, 042105 (2010); Phys. Rev. A 82, 069902 (E) (2010).
3. X. M. Lu, J. Ma, X. Zheng and X. Wang, Phys. Rev. A **83**, 012327 (2011).
4. Li. B, Z. X. Wang and S. M. Fei, Phys. Rev. A **83**, 022321 (2011).
5. D. Girolami and G. Adesso, Phys. Rev. A **83**, 052108 (2011).
6. Q. Chen, C. Zhang, S. Yu, X. X. Yi and C. H. Oh, Phys. Rev. A **84**, 042313 (2011).
7. ……

3、Results & Discussions

Considering a two - level system

s.p. energies $\varepsilon_1 = 0,\ \varepsilon_2 = 1$; degeneracies $\Omega_1 = \Omega_2 = \Omega$

strength of paring force $G_{k\bar{k}k'\bar{k}'} = \begin{cases} 4G_{ii} & (i = 1, 2) \\ 4G_{12} = 4G_{21} & \end{cases}$

$|\mathrm{BCS}\rangle$: direct product state of two-qubit with different modes

$\rightarrow$ Both entanglement and discord are vanishing.

$\rightarrow$ BCS volates symmetry and washes out quantumeness

PBCS

Computational bases of two-qubit Hilbert space

$\{|ij\rangle \equiv |00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

i, j correspond to qubit $A, B,$ respectively

3、Results & Discussions

For qubits *A*, *B*, there are three types of reduced density matrices, corresponding to three cases of the two level being occupied.

The meaning of ‘reduce’…

All the three cases are special X-states

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad \rho_{14} = \rho_{41} = 0$$

3、Results & Discussions

Type-1

$$\rho_{11}^{(1)} = C\sum_i \binom{\Omega_1 - 2}{i}\left(\frac{v_1^2}{u_1^2}\right)^i \binom{\Omega_2}{p-i}\left(\frac{v_2^2}{u_2^2}\right)^{p-i}, \quad C = u_1^{2\Omega_1}u_2^{2\Omega_2}$$

$$\rho_{22}^{(1)} = \rho_{33}^{(1)} = \rho_{23}^{(1)} = \rho_{32}^{(1)} = C\sum_i \binom{\Omega_1 - 2}{i-1}\left(\frac{v_1^2}{u_1^2}\right)^i \binom{\Omega_2}{p-i}\left(\frac{v_2^2}{u_2^2}\right)^{p-i},$$

$$\rho_{44}^{(1)} = C\sum_i \binom{\Omega_1 - 2}{i-2}\left(\frac{v_1^2}{u_1^2}\right)^i \binom{\Omega_2}{p-i}\left(\frac{v_2^2}{u_2^2}\right)^{p-i}$$

Type - 2 ……

Type - 3

$$\to \rho^{(1)}(p) = \rho^{(3)}(2\Omega - p), \text{ when } \Omega_1 = \Omega_2 = \Omega$$

3.1. Two-level case

$$\mathbb{C}(\rho) = \max\left\{0, 2\sqrt{\rho_{22}\rho_{33}} - 2\sqrt{\rho_{11}\rho_{44}}\right\},$$

$$\begin{aligned}\mathcal{I}(\rho) = {} & \rho_{11}\log_2\rho_{11} + \rho_{44}\log_2\rho_{44} \\ & + (\rho_{22}+\rho_{33})\log_2(\rho_{22}+\rho_{33}) - \{(\rho_{11}+\rho_{22})\log_2(\rho_{11}+\rho_{22}) \\ & + (\rho_{33}+\rho_{44})\log_2(\rho_{33}+\rho_{44}) + \text{terms with } 2 \rightleftarrows 3\},\end{aligned}$$

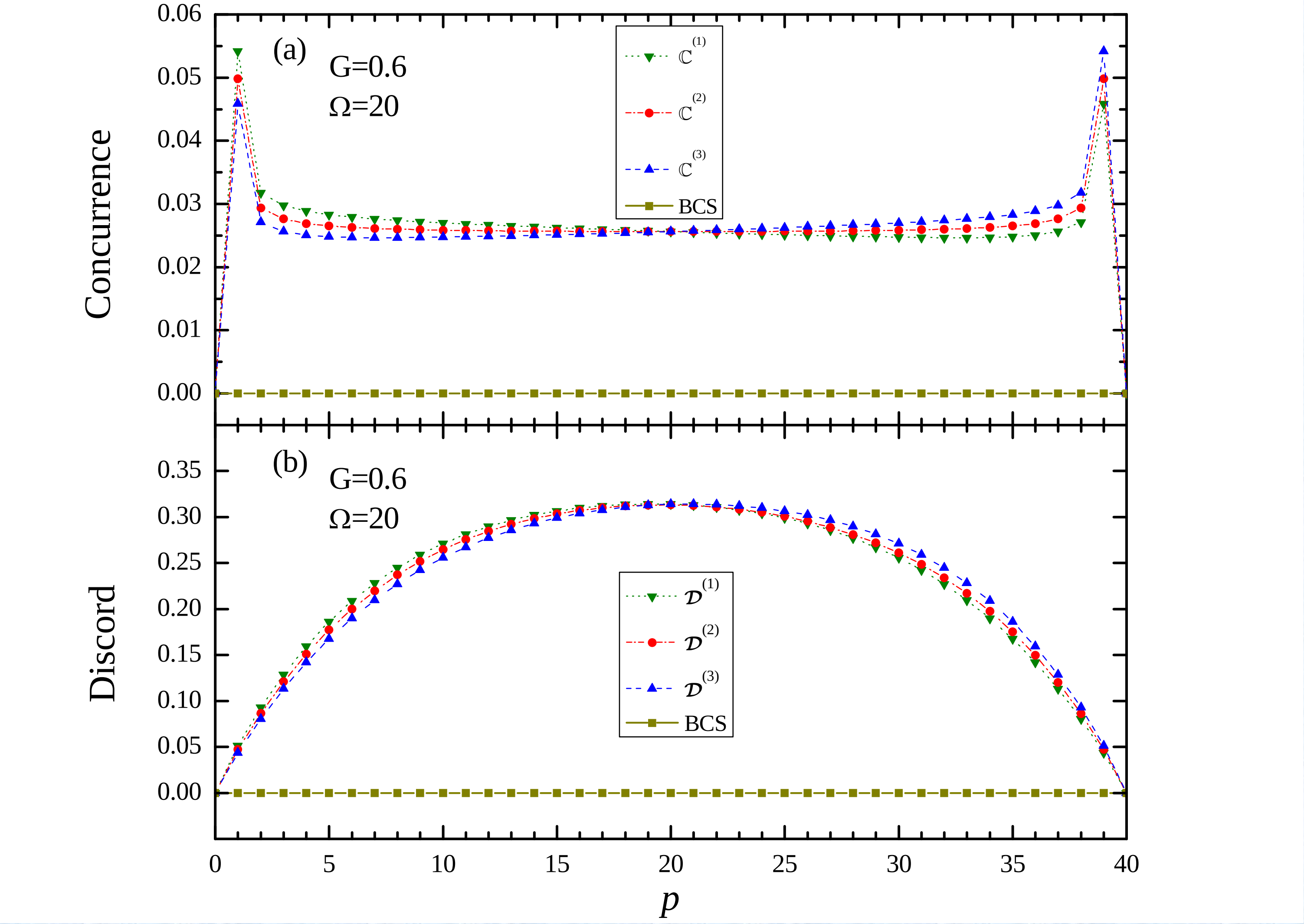
$$\mathcal{C}(\rho) = (\rho_{11}+\rho_{22})\log_2(\rho_{11}+\rho_{22}) - (\rho_{33}+\rho_{44})\log_2(\rho_{33}+\rho_{44}) - \min\{S_1, S_2\},$$

$$\begin{aligned}S_1 = {} & -\rho_{11}\log_2\frac{\rho_{11}}{\rho_{11}+\rho_{33}} - \rho_{33}\log_2\frac{\rho_{33}}{\rho_{11}+\rho_{33}} \\ & - \rho_{22}\log_2\frac{\rho_{22}}{\rho_{22}+\rho_{44}} - \rho_{44}\log_2\frac{\rho_{44}}{\rho_{22}+\rho_{44}},\end{aligned}$$

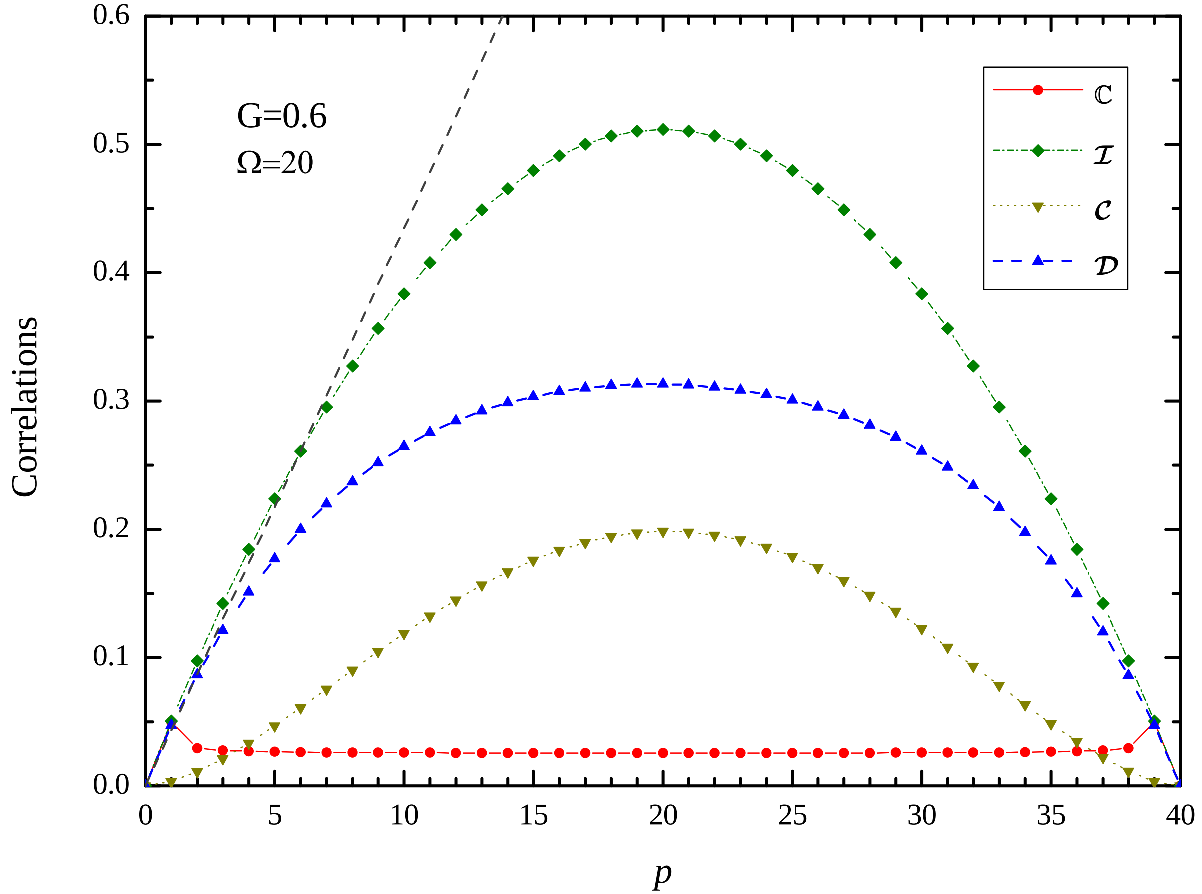
$$S_2 = -\frac{1-\theta}{2}\log_2\frac{1-\theta}{2} - \frac{1+\theta}{2}\log_2\frac{1+\theta}{2}, \ \theta = \sqrt{(\rho_{11}+\rho_{22}-\rho_{33}-\rho_{44}) + 4\rho_{22}\rho_{33}}$$

$$\mathcal{D}(\rho) = \mathcal{I}(\rho) - \mathcal{C}(\rho)$$

3.1. Two-level case



3.1. Two-level case

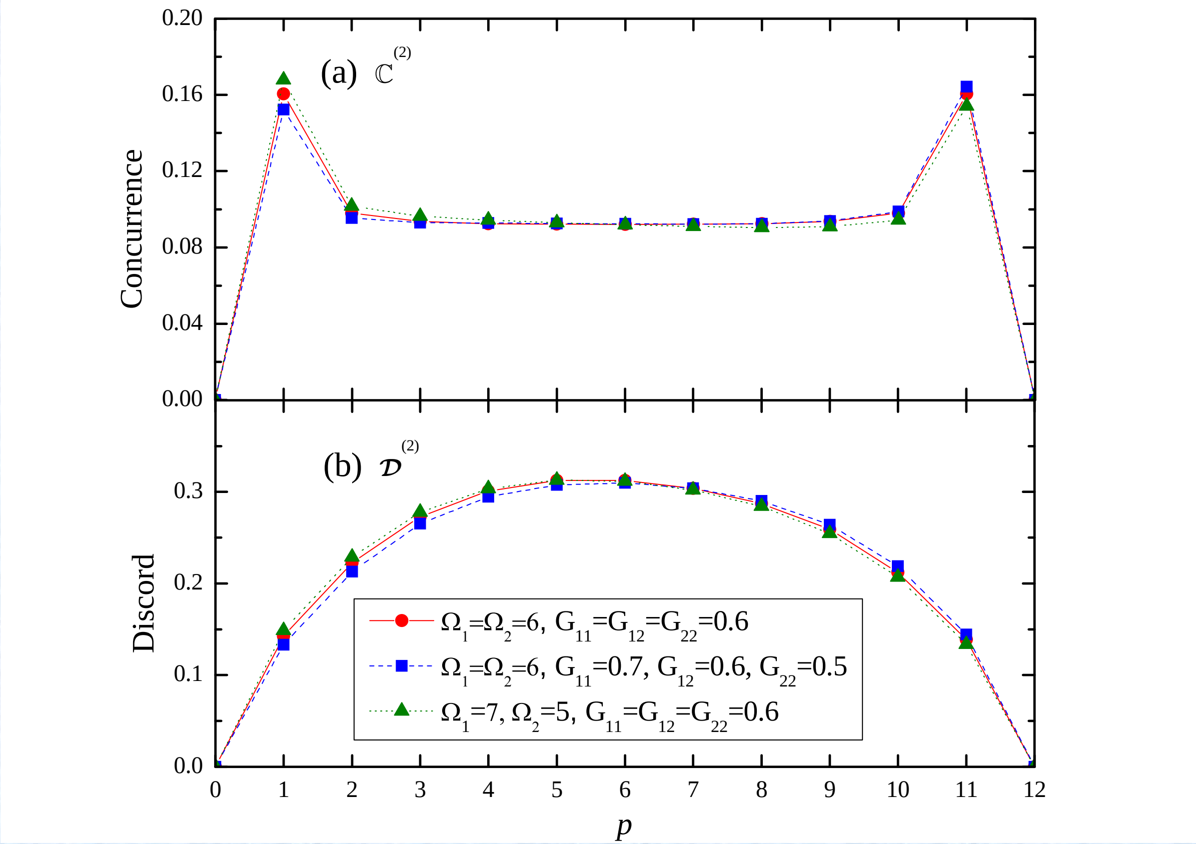


3.1. Results —— Two-level Case

- ➔ There's no big difference among three types; all have *p*-*h* symmetry, especially for type 2.
- ➔ When the number of particle-pair takes 1, concurrence jumps to a maximum from zero, then drops a little when p=2 and hardly changes with p afterwards.
  → intensive quantity
- ➔ Discord increases with particle pair, saturates at $p=\Omega$; when $p \ll \Omega$, discord varies linearly with *p.*
  → extensive quantity
- ➔ Total correlation behaves more linearly as varying vs p
- ➔ Discord is always larger than classical correlation.
  it's only valid in our case, not an universal rule.

3.1. Results —— Exact Solutions

Hamiltonian with paring force can be exactly diagonalized, which enables us to discuss the effects of the variations of G & Ω. It's hardly to do so with PBCS.



3.2. One-Level Case

Limit of two-level case, Hamiltonian has analytic solution

$$\rho = \frac{1}{\Omega(\Omega-1)}\begin{pmatrix} q(q-1) & 0 & 0 & 0 \\ 0 & pq & pq & 0 \\ 0 & pq & pq & 0 \\ 0 & 0 & 0 & p(p-1) \end{pmatrix}$$

$q = \Omega - p$ $\rightarrow$ number of hole-pair; G disappears

(pair) correlations rather than interactions determine the quantumness of a many-body system

3.2. One-Level Case

All correlations have analytic expressions, symmetric with $p \leftrightarrow q$

$$\mathbb{C}(\rho) = \frac{2}{\Omega(\Omega-1)}\left[pq - \sqrt{p(p-1)q(q-1)}\right]$$

$$\mathcal{I}(\rho) = \frac{p(p-1)}{\Omega(\Omega-1)}\log_2\frac{p(p-1)}{\Omega(\Omega-1)} + \frac{pq}{\Omega(\Omega-1)}\log_2\frac{pq}{\Omega(\Omega-1)} - \frac{2p}{\Omega}\log_2\frac{p}{\Omega}$$

$$+ \text{terms with } p \leftrightarrow q$$

$$\mathcal{C}(\rho) = \frac{1-\theta}{2}\log_2\frac{1-\theta}{2} + \frac{1+\theta}{2}\log_2\frac{1+\theta}{2} - \frac{p}{\Omega}\log_2\frac{p}{\Omega} - \frac{q}{\Omega}\log_2\frac{q}{\Omega}$$

$$\theta = \frac{1}{\Omega(\Omega-1)}\sqrt{(p-q)^2(\Omega-1)^2 + 4p^2q^2}$$

$$\mathcal{D}(\rho) = \mathcal{I}(\rho) - \mathcal{C}(\rho)$$

3.2. One-level Case——$\Omega >> 1$

$$\mathbb{C} \to 0,$$

$$\mathcal{I}_{\max} \to \frac{1}{2},$$

$$\mathcal{C}_{\max} \to \frac{3}{4}\log_2\frac{3}{4} \simeq 0.189,$$

$$\mathcal{D}_{\max} \to \frac{3}{2} - \frac{3}{4}\log_2 \simeq 0.311$$

When $p \ll \Omega$

$$\mathcal{I}(\rho) \simeq \frac{2p}{\Omega} \to \text{linearity}$$

$$\mathcal{C}(\rho) \simeq -\frac{p}{\Omega}\log_2\frac{p}{\Omega},$$

$$\mathcal{D}(\rho) \simeq \frac{2p}{\Omega} + \frac{p}{\Omega}\log_2\frac{p}{\Omega}$$

3.3. Applications to Realistic Nuclei

${}^{18}_{8}\mathrm{O}_{10}$, 0^+ states

Taking ${}^{16}_{8}\mathrm{O}_{8}$ as an inert core

Model space: $d_{5/2}s_{1/2}$, $d_{5/2}s_{1/2}d_{3/2}$

Residual interaction: δ potential

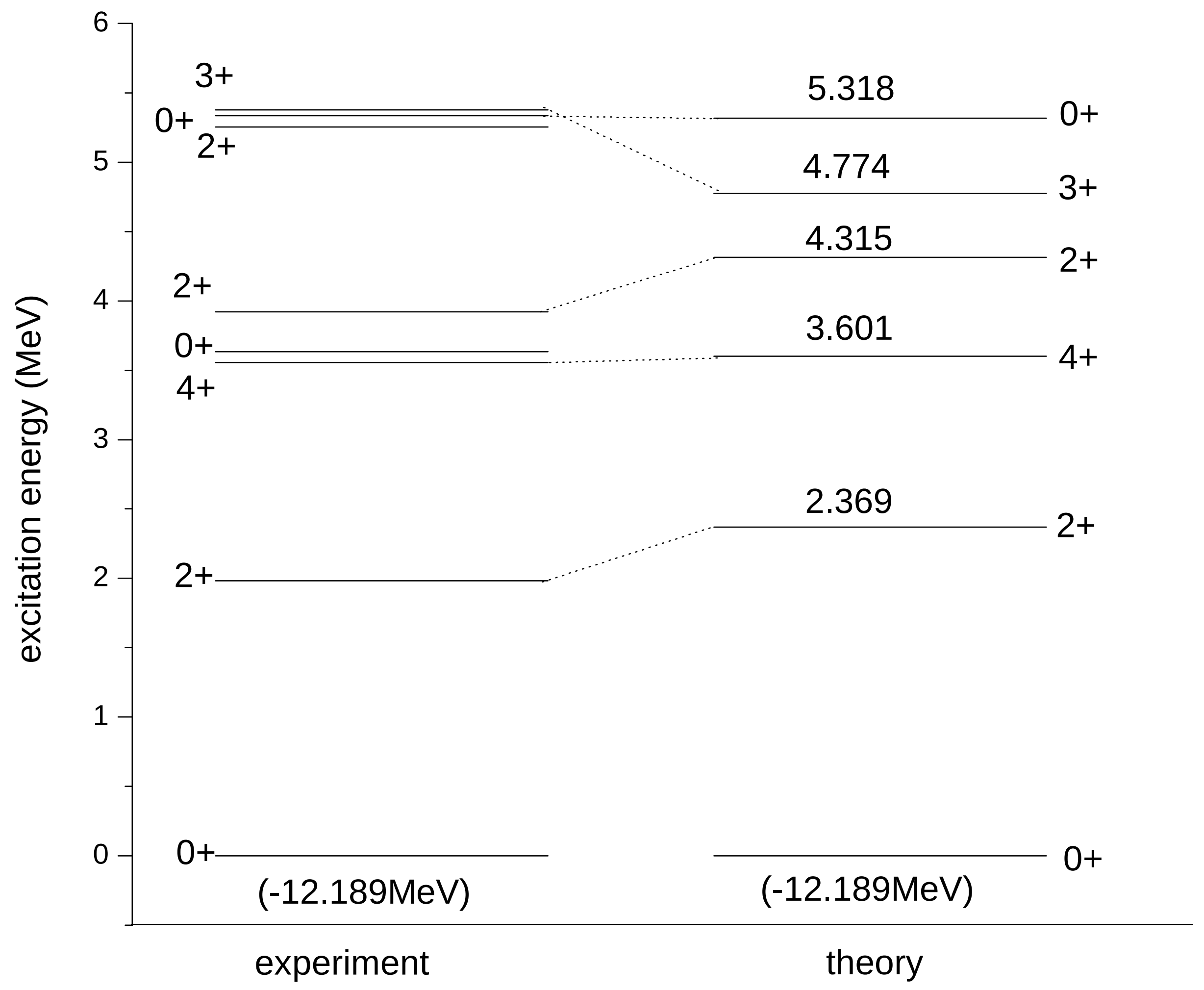
Fitting observed spectra $\rightarrow$ 0^+ wave function

which is the direct product of

$$d_{5/2,\pm 1/2}, d_{5/2,\pm 3/2}, d_{5/2,\pm 1/2}, s_{1/2,\pm 1/2}$$

R. D. Lawson, *Theory of the Nuclear Shell Model*, Clarendon Press, Oxford, 1980.

3.3. Applications to Realistic Nuclei



3.3. Applications to Realistic Nuclei

表 1　第一个 0^+ 态 $\left|\psi_{I=0}^{1}\right\rangle = 0.929\left(d_{5/2}\right)_{00}^{2} + 0.371\left(s_{1/2}\right)_{00}^{2}$ 的纠缠度

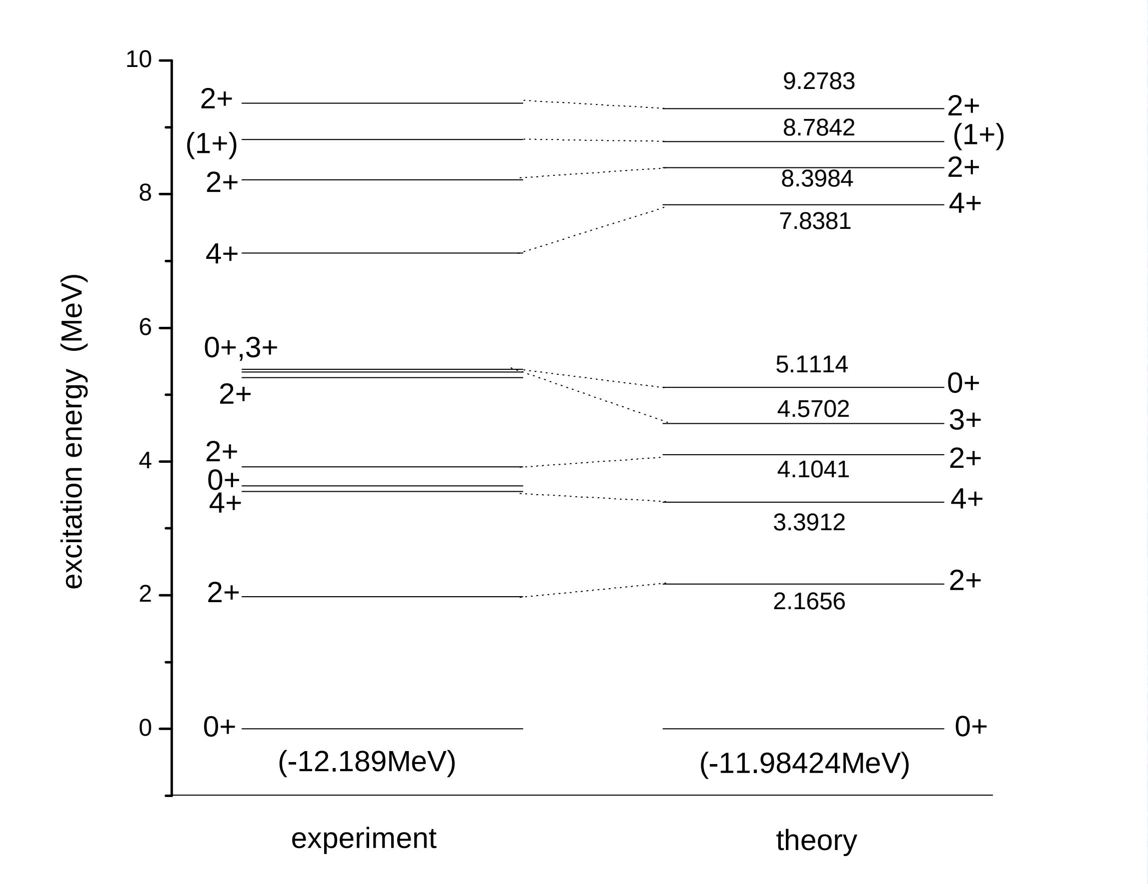
	negativity	concurrence	discord
$d_{5/2}$ 内对态	0.290	0.575	0.472
$d_{5/2}s_{1/2}$ 间对态	0.124	0.398	0.306
平均值	0.207	0.487	0.389

表 2　第二个 0^+ 态 $\left|\psi_{I=0}^{2}\right\rangle = -0.371\left(d_{5/2}\right)_{00}^{2} + 0.929\left(s_{1/2}\right)_{00}^{2}$ 的纠缠度

	negativity	concurrence	discord
$d_{5/2}$ 内对态	0.005	0.092	0.083
$d_{5/2}s_{1/2}$ 间对态	0.317	0.398	0.250
平均值	0.161	0.245	0.166

3.3. Applications to Realistic Nuclei

Model space: $d_{5/2}s_{1/2}d_{3/2}$



3.3. Applications to Realistic Nuclei

Model space: $d_{5/2}s_{1/2}d_{3/2}$

表 3 基态 $\left|\psi^1_{T=0}\right\rangle = 0.912(d_{5/2})^2_{00} + 0.358(s_{1/2})^2_{00} + 0.199(d_{3/2})^2_{00}$ 的纠缠度

	negativity	concurrence	discord
$d_{5/2}$ 内对态	0.266	0.554	0.453
$d_{5/2}s_{1/2}$ 间对态	0.110	0.377	0.314
$d_{5/2}d_{3/2}$ 间对态	0.015	0.148	0.095
$s_{1/2}d_{3/2}$ 间对态	0.006	0.101	0.077
$d_{3/2}$ 内对态	0.001	0.040	0.037
平均值	0.082	0.262	0.204

3.3. Applications to Realistic Nuclei

Model space: $d_{5/2}s_{1/2}d_{3/2}$

表 4 第二个 0^+ 态 $|\psi^2_{I=0}\rangle = 0.374(d_{5/2})^2_{00} - 0.926(s_{1/2})^2_{00} - 0.049(d_{3/2})^2_{00}$ 的纠缠度

	negativity	concurrence	discord
$d_{5/2}$ 内对态	0.005	0.093	0.085
$d_{5/2}s_{1/2}$ 间对态	0.316	0.400	0.176
$d_{5/2}d_{3/2}$ 间对态	0.0001	0.015	0.007
$s_{1/2}d_{3/2}$ 间对态	0.014	0.064	0.012
$d_{3/2}$ 内对态	0.0	0.002	0.001
平均值	0.066	0.113	0.057

3.3. Applications to Realistic Nuclei

† $^{18}_{10}\mathrm{Ne}_8$: mirror nucleus of $^{18}_{10}\mathrm{O}_8$

Both entanglement & discord are charge-symmetric

† $^{58}_{28}\mathrm{Ni}_{30}$, model space: $p_{3/2}\, f_{5/2}\, p_{1/2}\ \cdots$

贾超，吉林大学硕士学位论文，2012年6月

4. Conclusions

- ■ The system's quantumness is reestablished with the restoration of the symmetry (particle number conservation);
- ■ Entanglement and discord are the distinguish aspects of quantum correlations in a quantum system. The former behaves as a intensive quantity, while the latter extensive quantity.
- ▌Quantum correlations are not sensitive to the strength of pairing force and energy degeneracy, but sensitive to the way the particles correlate.
- □ Systematical study on a series of realistic nuclei.
- □ The different features of entanglement and discord may have some applications in quantum computation and quantum communication


Thanks for your attention