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Relativistic extension of CSM for the resonant states in deformed nuclei

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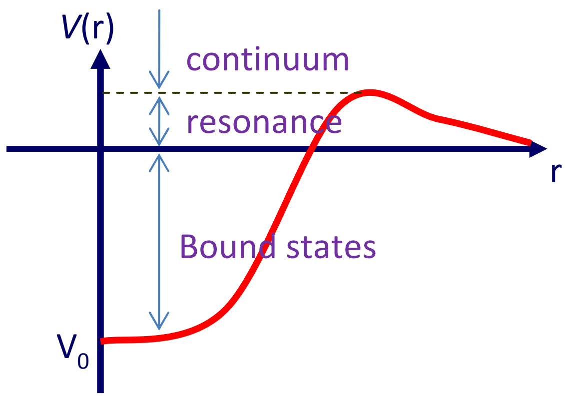
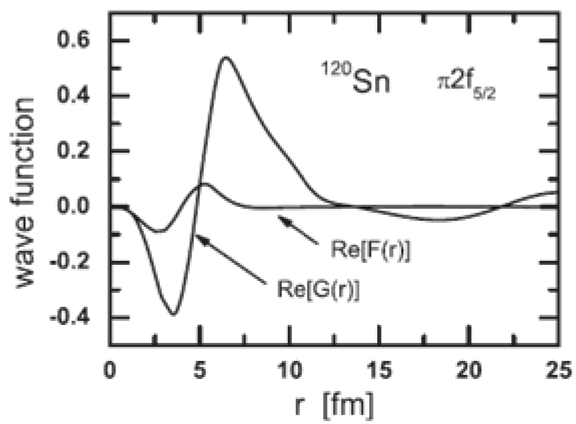
The significance of resonant states

➢ When the particle energy meets the condition $0<E<V_{max}$, the particle lies in resonant state. And the wave function of resonant state oscillates even at large radius r.





➢ Resonant states play an important role not only in nuclear physics, but also in many branches of science, such as atomic, molecular, and nanophysics.

G. Gamow 1928 ZPA, T. Sommerfeld 1998 PRL, N. Moiseyev 1979 PRA,M. Bylicki 2005 PRB

➢ The resonances has been thought to be the cause of some exotic nuclear phenomena, such as halo, giant halo, and deformed halo.

I. Tanihata 1985 PRL, J. Meng 1996 PRL, W. Pöschl 1997 PRL, N. Sandulescu 2000 PRC; J. Meng 1998 PRL, Y. Zhang 2012 PRC, I. Hamamoto 2010 PRC(R), S.G. Zhou 2010 PRC(R)

➢ The resonances in the continuum play an important role in the description of the nuclear dynamical processes, such as the collective giant resonances.

P. Curutchet 1989 PRC, L.G. Cao 2002 PRC

Methods for exploring resonant states

- Several methods based on scattering theory have been employed to study resonant states, such as

  R-matrix theory E. P. Wigner 1947 Phys. Rev., G. M. Hale 1987 PRL

  K-matrix theory J. Humblet 1991 PRC

  S-matrix method J. R. Taylor 1972

- Some bound-state-like methods have been developed, including

  The real stabilization method (RSM) A.U. Hazi 1970 PRA

  The analytic continuation in the coupling constant (ACCC) method

  V.I. Kukuli 1989 Kluwer Academic

  The complex scaling method (CSM) Y.K. Ho 1983 Phys. Rep.

- The development of these methods in the relativistic framework

  RMF-ACCC S.C. Yang 2001 CPL, S.S. Zhang 2004 PRC

  J.Y. Guo 2005 PRC, J.Y. Guo 2006 PRC, S. S. Zhang 2012 PRC

  RMF-RSM L. Zhang 2008 PRC, Z. Z. Zhang 2010 MPLA

  RMF-CSM J.Y. Guo 2010 PRC, Q. Liu 2013 PRA

  Jost function approach B.N. Lu 2012 PRL, B.N. Lu 2013 PRC

Methods for exploring resonant states

- The halo phenomenon in ^{11}Li has been discovered in 1985, the structure of exotic nuclei, especially those close to the neutron or proton drip line, has attracted wide attention in theory and experiment. I. Tanihata 1985 PRL

- The early works mainly focus on the light exotic nuclei. Recent experiments reveal the existence of exotic phenomena like ^{31}Ne and ^{37}Mg, which have aroused increasing interests in the study of exotic nuclei.
T. Nakamura 2009 PRL, T. Nakamura 2014 PRL, N. Kobayashi 2014 PRL

- The ACCC, RSM and the coupling-channel method have been applied in the deformed nuclei to investigate the resonant states and bound states.
G. Gattapan 2000 PRC, S. S. Zhang 2014 PLB, K. Hagino 2004 NPA, Z. P. Li 2010 PRC

- We have developed a formalism of describing the resonant states in deformed nuclei by CSM in non-relativistic framework.
Q. Liu 2012 PRC

Present work：

- We will extend the CSM to an axially symmetrical relativistic framework, and examine its applicability and efficiency for the resonant states in deformed nuclei.

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Theoretical framework

The Dirac equation for a particle moving in a repulsive vector potential $V(\vec{\mathbf{r}})$ and an attractive scalar potential $S(\vec{\mathbf{r}})$ can be written as

$$[\vec{\alpha}\bullet\vec{p}+V(\vec{\mathbf{r}})+\beta(M+S(\vec{\mathbf{r}}))]\psi_i(\vec{\mathbf{r}})=\varepsilon_i\psi_i(\vec{\mathbf{r}}) \qquad (1)$$

Here, only the axially quadruple deformation is considered, $V(\vec{\mathbf{r}})$ and $S(\vec{\mathbf{r}})$ are taken as

$$\begin{cases} V(\vec{r})=V_0 f(r)-\beta_2 V_0 k(r)Y_{20}(\vartheta,\varphi) \\ S(\vec{r})=S_0 f(r)-\beta_2 S_0 k(r)Y_{20}(\vartheta,\varphi) \end{cases}$$

The starting point of CSM is a transformation with the scaling operator U(θ)

$$H \Longrightarrow H_\theta=U(\theta)HU^{-1}(\theta)$$

$$\psi(\vec{r}) \Longrightarrow \psi_\theta=U(\theta)\psi(\vec{r})$$

where $U(\theta)=\begin{pmatrix} e^{i\theta\hat{S}} & 0 \\ 0 & e^{i\theta\hat{S}} \end{pmatrix}$

The complex scaled radial Dirac equation for nucleon is arrived as

$$H_\theta\psi_\theta(\vec{r})=\varepsilon_\theta\psi_\theta(\vec{r}) \qquad (2)$$

Theoretical framework

The solution of Eq.(2) can be obtained by the harmonic oscillator basis expansion method. The large and small components of the Dirac spinors $f_\theta(\vec{r})$ and $g_\theta(\vec{r})$ can be expanded in terms of a set of orthogonal normalization functions $\Phi_\alpha(\vec{r},s)$ as

$$\begin{cases} f_\theta(\vec{r}) = \sum_{\alpha=1}^{\alpha_{\max}} f_\alpha(\theta)\Phi_\alpha(\vec{r},s) \\ g_\theta(\vec{r}) = \sum_{\tilde{\alpha}=1}^{\tilde{\alpha}_{\max}} g_{\tilde{\alpha}}(\theta)\Phi_{\tilde{\alpha}}(\vec{r},s) \end{cases} \qquad (3)$$

with $\Phi_\alpha(\vec{r},s) = R_{nl}(r)Y_{lm}(\vartheta,\varphi)\chi_{m_s}(s)$.

The corresponding complex scaled Dirac Hamiltonian is

$$H_\theta = \begin{pmatrix} A_{\alpha'\alpha} & B_{\alpha'\tilde{\alpha}} \\ B_{\tilde{\alpha}'\alpha} & C_{\tilde{\alpha}'\tilde{\alpha}} \end{pmatrix} \quad \text{with} \quad \begin{cases} A_{\alpha'\alpha} = \int d\vec{r}\,\Phi^*_{\alpha'}(\vec{r},s)A_\theta\Phi_\alpha(\vec{r},s) \\ B_{\alpha'\tilde{\alpha}} = \int d\vec{r}\,\Phi^*_{\alpha'}(\vec{r},s)(iB_\theta)\Phi_{\tilde{\alpha}}(\vec{r},s) \\ C_{\tilde{\alpha}'\tilde{\alpha}} = \int d\vec{r}\,\Phi^*_{\tilde{\alpha}'}(\vec{r},s)C_\theta\Phi_{\tilde{\alpha}}(\vec{r},s) \end{cases}$$

Take the nucleus with A=31 as an example, we obtain the single-particle levels and compare them with the results by coupled channel method.

Numerical details:

In this work, we adopt the Woods-Saxon type potential

$$f(r) = 1/\{1+\exp[(r-R)]/a\}$$

The corresponding parameters are chosen as

V_0=350MeV and S_0=-405MeV, a=0.67fm,

the radius $R=r_0A^{1/3}$ with r_0=1.27fm.

Z. P. Li 2010 PRC

Harmonic oscillator shells N=60.

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Results and Discussions

M. Shi, Q. Liu, Z. M. Niu, J. Y. Guo, Phys. Rev. C 90, 034319 (2014)

To display all the states in the complex energy plane.

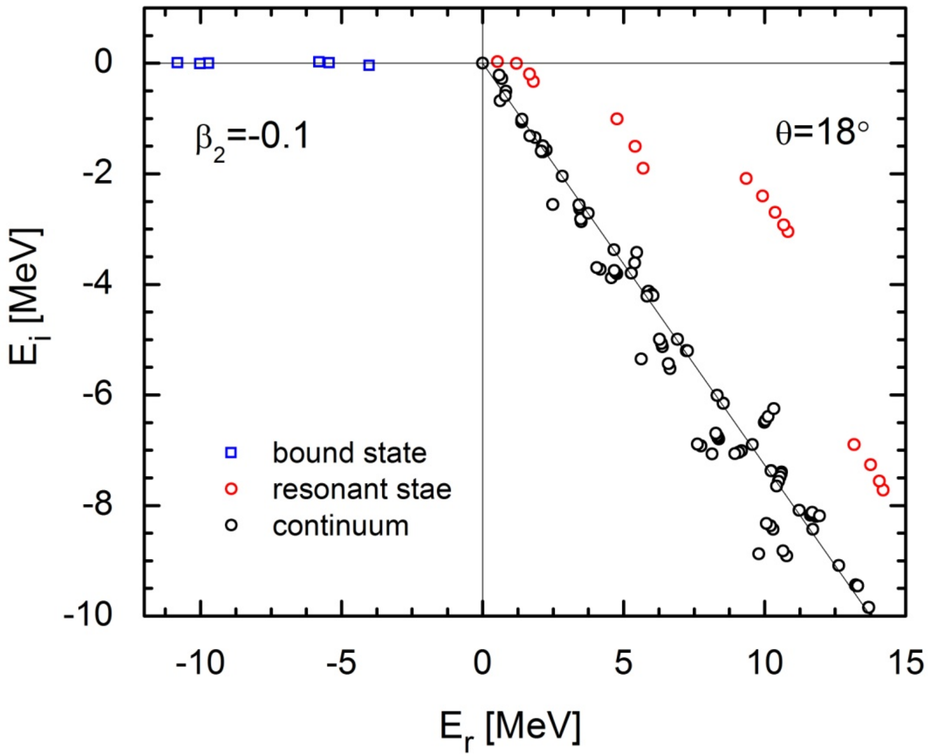


Fig.1 The eigenvalues of the complex scaled Hamiltonian H_θ.

- ➢The bound state (blue square) don't change with angle θ .
- ➢The continuum state (black circle) rotates by the angle.
- ➢The resonant state (red circle) nearly don't change with angle θ once it is separated from continuum state.

Results and Discussions

To show how the resonant states become isolated form the continuous spectrum by complex rotation.

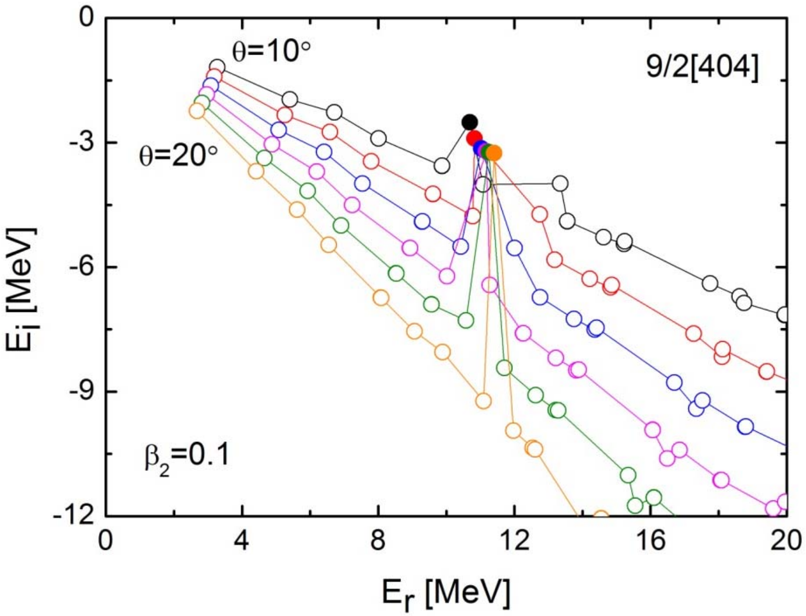


Fig.2 The variation of the eigenvalues of H_θ with θ for the states with $\Omega^\pi = 3/2^-$.

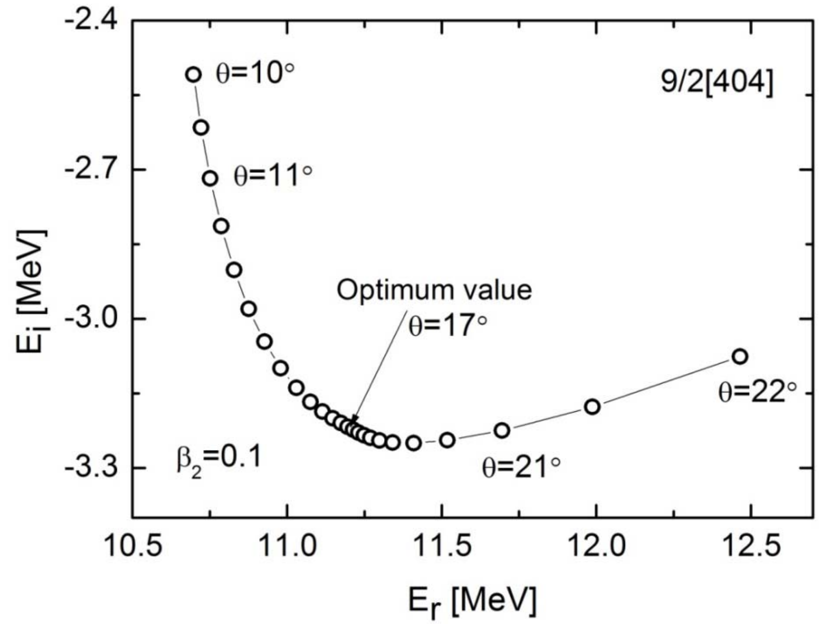


Fig.3 The θ -trajectories for the resonant state 3/2[301].

- ➢Since there exist approximations in the realistic calculations, the resonance position appears movement with the variation of θ .
- ➢The arrow marks the position of the resonance parameters in the optimum value for the resonant state under consideration.

Results and Discussions

To recognize the relationship between the resonant states and the deformation parameter β_2 of this model.

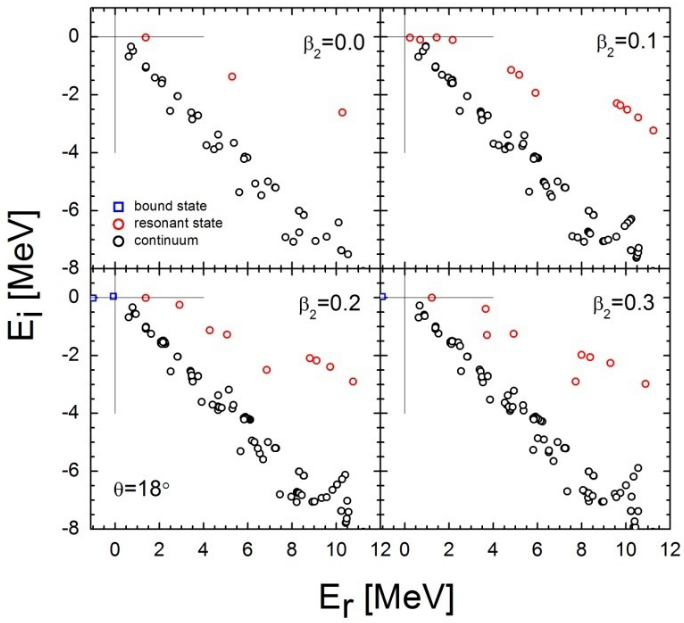


Fig.4 The eigenvalues of H_θ with the deformation parameter β_2.

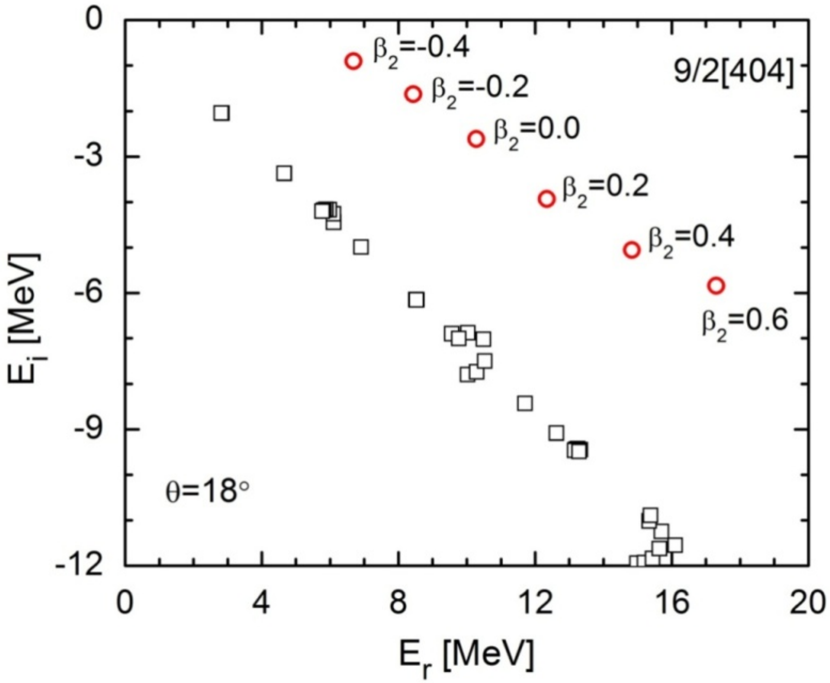


Fig.5 The movement of the resonance position with the deformation parameter β_2

- When $\beta_2 \neq 0$, the 3 resonant states are split into 12 resonant states.
- The similar phenomenon also appears in the oblate side.
- With the development of deformation, the resonance position appears remarkable movement in the complex energy plane, while the change of the continuum is negligent.

Results and Discussions

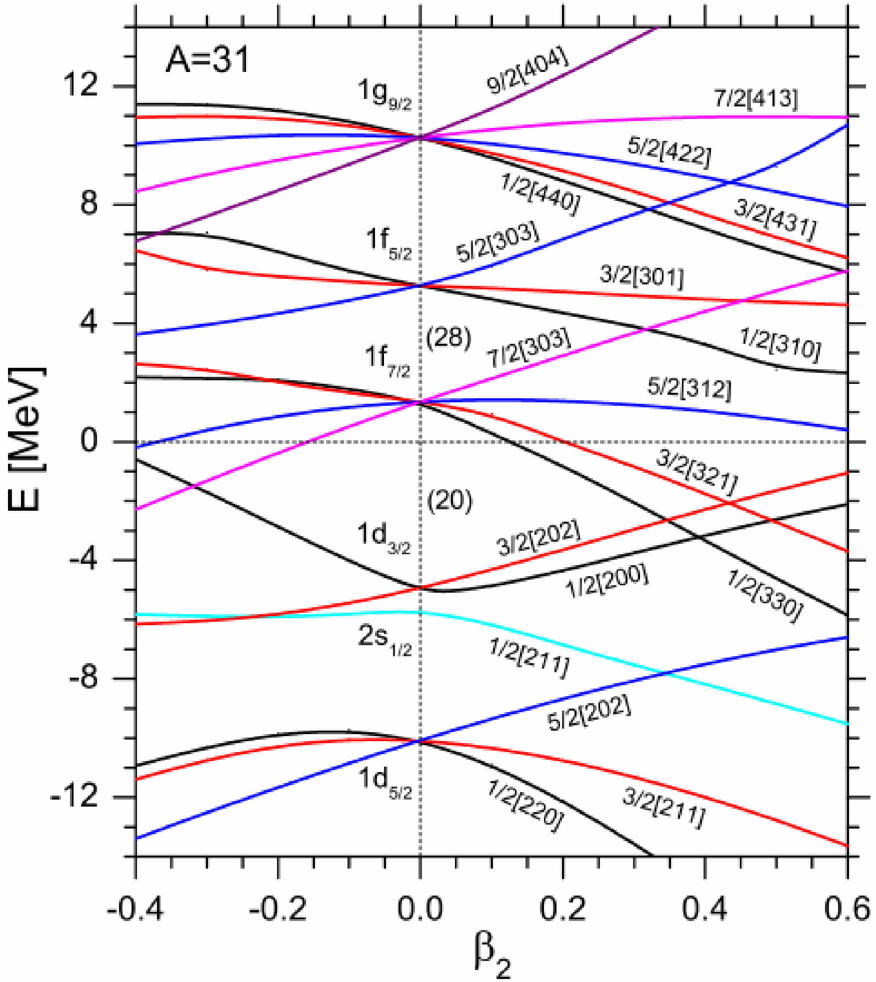
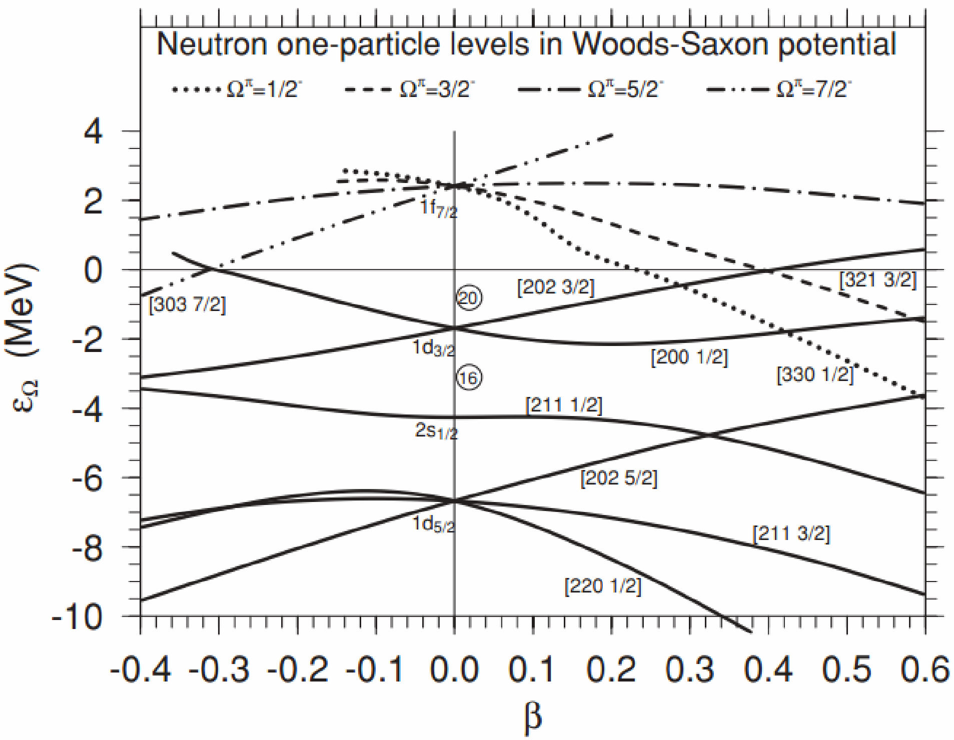


Fig.6 The single-particle levels in the nucleus with A=31 as a function of the quadruple deformation parameter β_2.



- ➢The present calculations by complex scaling method have presented richer results for the resonant levels.
- ➢These indicate the relativistic extension of complex scaling method is applicable and efficient in describing the resonant states for deformed nuclei.

Contents

Summary and Perspective

Summary:

- The theoretical formalism of the relativistic extension of CSM for the resonant states in deformed nuclei is presented.
- The efficient and applicable of the model is tested.

Perspective:

- The RMF-CSM method is applied to study the resonances for deformed nuclei in the relativistic framework.

Thank you！